

Week 14

10.10) (10.11)

$$\frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^r (x_{ij} - x_{i'j})^2 =$$

$$2 \sum_{i \in C_k} \sum_{j=1}^r (x_{ij} - \bar{x}_{kj})^2$$

$$\frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^r [(x_{ij} - \bar{x}_{kj}) - (x_{i'j} - \bar{x}_{kj})]^2$$

$$= \frac{|C_k|}{|C_k|} \sum_{i \in C_k} \sum_{j=1}^r (x_{ij} - \bar{x}_{kj})^2 + \frac{|C_k|}{|C_k|} \sum_{i' \in C_k} \sum_{j=1}^r (x_{i'j} - \bar{x}_{kj})^2$$

$$- \frac{2}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^r (x_{ij} - \bar{x}_{kj})(x_{i'j} - \bar{x}_{kj})$$

$$= 2 \sum_{i \in C_k} \sum_{j=1}^r (x_{ij} - \bar{x}_{kj})^2 \quad \square$$