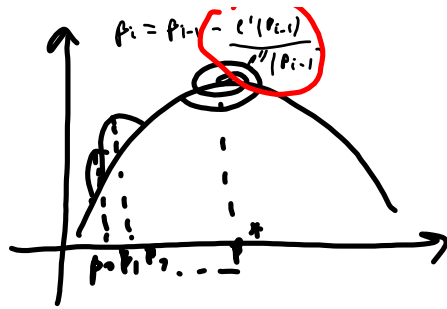




$$l(p + \Delta) = \frac{l(p)\Delta^0}{0!} + \frac{l'(p)\Delta^1}{1!} + \frac{l''(p)\Delta^2}{2!} + \dots$$

$$\frac{\partial l(p + \Delta)}{\partial \Delta} = 0 + l'(p)$$

$$\Rightarrow \Delta = \frac{-l'(p)}{l''(p)}$$

$$\begin{aligned} Y &\sim \text{Bernoulli} | p(x, \beta) = p(x, \beta)^Y (1 - p(x, \beta))^{1-Y} \\ X_1, \dots, X_n, Y_1, \dots, Y_n \\ \ell(\beta) &= \log p(Y_1, \dots, Y_n | X_1, \dots, X_n) = \sum_{i=1}^n Y_i \log p(X_i; \beta) + \\ &\quad + (1 - Y_i) \log (1 - p(X_i; \beta)) \\ &= \sum_{i=1}^n Y_i \log \frac{e^{X_i^T \beta}}{1 + e^{X_i^T \beta}} + (1 - Y_i) \log \frac{1}{1 + e^{X_i^T \beta}} \\ &= \sum_{i=1}^n Y_i \log \frac{e^{X_i^T \beta}}{1 + e^{X_i^T \beta}} + (1 - Y_i) \log \frac{1}{1 + e^{X_i^T \beta}} \\ &= \sum_{i=1}^n Y_i [X_i^T \beta - \log(1 + e^{X_i^T \beta})] + (1 - Y_i) \log(1 + e^{X_i^T \beta}) \\ &= \sum_{i=1}^n Y_i (X_i^T \beta - \log(1 + e^{X_i^T \beta})) + \log(1 + e^{X_i^T \beta}) \\ &= \sum_{i=1}^n Y_i (X_i^T \beta - \log(1 + e^{X_i^T \beta})) = \sum_{i=1}^n Y_i (X_i^T \beta) - \log(1 + e^{X_i^T \beta}) \end{aligned}$$

$$\begin{aligned} \frac{\partial \ell(\beta)}{\partial \beta} &= \sum_{i=1}^n Y_i X_i^T - \left( \frac{1}{1 + e^{X_i^T \beta}} \right) e^{X_i^T \beta} X_i^T = \\ &= \sum_{i=1}^n X_i^T \left( Y_i - \frac{e^{X_i^T \beta}}{1 + e^{X_i^T \beta}} \right) = \sum_{i=1}^n X_i^T (Y_i - p(X_i, \beta)) \end{aligned}$$

$$\frac{\partial \ell(\beta)}{\partial \beta \partial \beta^T} = - \sum_{i=1}^n X_i X_i^T p(X_i; \beta) (1 - p(X_i; \beta))$$