

$$\hat{f}(x_0) = \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) Y_i}{\sum_{i=1}^N K_\lambda(x_0, x_i)}, \quad Y_i = f(x_i) + \varepsilon_i, \quad E(\varepsilon_i) = 0$$

$$\rightarrow E(Y_i) = f(x_i)$$

$$\text{Var}(Y_i) = \text{Var}(\varepsilon_i) = \sigma_\varepsilon^2$$

We have:

$$E(\hat{f}(x_0)) = E\left(\frac{\sum_{i=1}^N K_\lambda(x_0, x_i) Y_i}{\sum_{i=1}^N K_\lambda(x_0, x_i)}\right)$$

$$= \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) E(Y_i)}{\sum_{i=1}^N K_\lambda(x_0, x_i)}$$

$$= \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) f(x_i)}{\sum_{i=1}^N K_\lambda(x_0, x_i)}$$

$$= \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) (-f(x_i) - f(x_0) + f(x_0))}{\sum_{i=1}^N K_\lambda(x_0, x_i)}$$

$$= \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) (-f(x_i) - f(x_0))}{\sum_{i=1}^N K_\lambda(x_0, x_i)} + \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) f(x_0)}{\sum_{i=1}^N K_\lambda(x_0, x_i)} = f(x_0) \frac{\sum_{i=1}^N K_\lambda(x_0, x_i)}{\sum_{i=1}^N K_\lambda(x_0, x_i)}$$

$$= f(x_0) + \frac{\sum_{i=1}^N K_\lambda(x_0, x_i) (f(x_i) - f(x_0))}{\sum_{i=1}^N K_\lambda(x_0, x_i)}$$

$$\text{Var}(\hat{f}(x_0)) = \text{Var}\left(\frac{\sum_{i=1}^N K_\lambda(x_0, x_i) Y_i}{\sum_{i=1}^N K_\lambda(x_0, x_i)}\right)$$

$$= \frac{\sum_{i=1}^N K_\lambda(x_0, x_i)^2 \text{Var}(Y_i)}{\left(\sum_{i=1}^N K_\lambda(x_0, x_i)\right)^2}$$

$$= \sigma_\varepsilon^2 \sum_{i=1}^N \frac{K_\lambda(x_0, x_i)^2}{\left(\sum_{i=1}^N K_\lambda(x_0, x_i)\right)^2} = \sigma_\varepsilon^2 \sum_{i=1}^N w_{i,0}^2$$

$$D(\theta) = \overset{1 \times N}{y - B\theta}^T \overset{N \times N}{W(x_0)} \overset{N \times 1}{(y - B\theta)}$$

$$= y^T W(x_0) y - y^T W(x_0) B \theta - \underbrace{\theta^T B^T W(x_0) y}_{= y^T W(x_0) B \theta} + \theta^T B^T W(x_0) B \theta$$

$$= y^T W(x_0) y - 2 y^T W(x_0) B \theta + \theta^T B^T W(x_0) B \theta$$

$$\rightarrow \frac{\partial D(\theta)}{\partial \theta} = -2 B^T W(x_0) y + \left( B^T W(x_0) B + \underbrace{(B^T W(x_0) B)^T}_{B^T W(x_0) B} \right) \theta$$

$$= -2 B^T W(x_0) y + 2 B^T W(x_0) B \theta = 0$$

$$\rightarrow B^T W(x_0) B \theta = B^T W(x_0) y$$

$$\rightarrow \theta = (B^T W(x_0) B)^{-1} B^T W(x_0) y$$