

Deviance :

$$D = -2 \log(\hat{p}_1, \dots, \hat{p}_{K|\Pi|})$$

Let $z_{ik} = I(y_i = k)$, $i = 1, \dots, N$, $k = 1, \dots, K$

$$\begin{aligned} l(\hat{p}_1, \dots, \hat{p}_{K|\Pi|}) &= \log \left(\prod_{i=1}^N \prod_{k=1}^K \hat{p}_k(x_i)^{z_{ik}} \right) \\ &= \sum_{i=1}^N \sum_{k=1}^K z_{ik} \log(\hat{p}_k(x_i)) \\ &= \sum_{i=1}^N \sum_{k=1}^K z_{ik} \sum_{n=1}^{|\Pi|} I(x_i \in R_n) \log(\hat{p}_{nk}) \\ &= \sum_{n=1}^{|\Pi|} \sum_{k=1}^K \log(\hat{p}_{nk}) \sum_{i=1}^N z_{ik} I(x_i \in R_n) \\ &= \sum_{n=1}^{|\Pi|} \sum_{k=1}^K \log(\hat{p}_{nk}) \sum_{(i: x_i \in R_n)} z_{ik} \end{aligned}$$

Recall that

$$\hat{p}_{nk} = \frac{1}{N_n} \sum_{\{i: x_i \in R_n\}} \overbrace{I(y_i = k)}^{z_{ik}} = \frac{1}{N_n} \sum_{\{i: x_i \in R_n\}} z_{ik}$$

$$\rightarrow \sum_{\{i: x_i \in R_n\}} z_{ik} = N_n \hat{p}_{nk}$$

$$\begin{aligned} \rightarrow l(\hat{p}_1, \dots, \hat{p}_{K|\Pi|}) &= \sum_{n=1}^{|\Pi|} \sum_{k=1}^K \log(\hat{p}_{nk}) \cdot N_n \hat{p}_{nk} \\ &= \sum_{n=1}^{|\Pi|} N_n \underbrace{\sum_{k=1}^K \hat{p}_{nk} \log(\hat{p}_{nk})}_{Q_n(T)} = \sum_{n=1}^{|\Pi|} N_n Q_n(T) \end{aligned}$$