

Let $V_1, \dots, V_B$ be identically distributed samples with $\text{Var}(V_i) = \sigma^2$, $i=1, \dots, B$, and

$\text{Corr}(V_i, V_j) = \rho$ $\forall i \neq j$. We have:

$$\begin{aligned} \text{Var}\left(\frac{1}{B} \sum_{i=1}^B V_i\right) &= \frac{1}{B^2} \left(\sum_{i=1}^B \text{Var}(V_i) + 2 \sum_{i=1}^{B-1} \sum_{j=i+1}^B \text{Cov}(V_i, V_j) \right) \\ &= \frac{1}{B^2} \left(\underbrace{\sum_{i=1}^B \sigma^2}_{= B\sigma^2} + 2 \sum_{i=1}^{B-1} \sum_{j=i+1}^B \underbrace{\sigma \cdot \sigma \cdot \text{Corr}(V_i, V_j)}_{\rho} \right) \\ &\quad \underbrace{\hspace{10em}}_{\frac{B(B-1)}{2} \cdot \rho \cdot \sigma^2} \end{aligned}$$

$$= \frac{1}{B} \left(\sigma^2 + 2 \cdot \frac{B-1}{2} \rho \sigma^2 \right)$$

$$= \frac{1}{B} \left(\sigma^2 + B\rho\sigma^2 - \rho\sigma^2 \right)$$

$$= \frac{1-\rho}{B} \sigma^2 + \rho \sigma^2$$