

The law of double expectation:

$$E(X) = E(E(X|Z))$$

$$\hat{G}(x) = \arg \min_g \sum_{h=1}^K \mathbb{I}(g \neq h) P(G=h | X=x)$$

$$= \arg \min_g \sum_{h=1}^K (1 - \mathbb{I}(g=h)) P(G=h | X=x)$$

$$= \arg \min_g \underbrace{\sum_{h \neq g} P(G=h | X=x)}$$

$$= 1 - P(G=g | X=x)$$

$$= \arg \min_g (1 - P(G=g | X=x))$$

$$= \arg \max_g P(G=g | X=x)$$