

We have :

$$l(t, \lambda) = \sum_{i=1}^N (y_i \underline{N}(x_i)^T \underline{\theta} - \log(1 + e^{\underline{N}(x_i)^T \underline{\theta}})) - \frac{1}{2} \lambda \underline{\theta}^T \underline{\Omega} \underline{\theta}$$

We get :

$$\begin{aligned} \frac{\partial l(t, \lambda)}{\partial \underline{\theta}} &= \sum_{i=1}^N \left(y_i \underline{N}(x_i) - \frac{e^{\underline{N}(x_i)^T \underline{\theta}}}{1 + e^{\underline{N}(x_i)^T \underline{\theta}}} \cdot \underline{N}(x_i) \right) \\ &\quad - \frac{1}{2} \lambda (\underline{\Omega} + \underline{\Omega}^T) \underline{\theta} \\ &= \sum_{i=1}^N \left(y_i \underline{N}(x_i) - \underbrace{\frac{e^{\underline{N}(x_i)^T \underline{\theta}}}{1 + e^{\underline{N}(x_i)^T \underline{\theta}}} \cdot \underline{N}(x_i)}_{p(y_i=1|x_i) = p(x_i)} \right) - \lambda \underline{\Omega} \underline{\theta} \\ &= \sum_{i=1}^N (y_i - p(x_i)) \underline{N}(x_i) - \lambda \underline{\Omega} \underline{\theta} \\ &= \underline{N}^T (\underline{y} - \underline{p}) - \lambda \underline{\Omega} \underline{\theta} \end{aligned}$$

Further,

$$\begin{aligned} \frac{\partial^2 l(t, \lambda)}{\partial \underline{\theta} \partial \underline{\theta}^T} &= - \sum_{i=1}^N \frac{e^{\underline{N}(x_i)^T \underline{\theta}} (1 + e^{\underline{N}(x_i)^T \underline{\theta}}) \underline{N}(x_i) - e^{\underline{N}(x_i)^T \underline{\theta}} e^{\underline{N}(x_i)^T \underline{\theta}} \underline{N}(x_i)}{(1 + e^{\underline{N}(x_i)^T \underline{\theta}})^2} \\ &\quad - \lambda \underline{\Omega} \\ &= - \sum_{i=1}^N \frac{e^{\underline{N}(x_i)^T \underline{\theta}} + \cancel{2e^{\underline{N}(x_i)^T \underline{\theta}}} - \cancel{e^{\underline{N}(x_i)^T \underline{\theta}}}}{(1 + e^{\underline{N}(x_i)^T \underline{\theta}})^2} \cdot \underline{N}(x_i) \underline{N}(x_i)^T \\ &\quad - \lambda \underline{\Omega} \\ &= - \sum_{i=1}^N \underbrace{\frac{e^{\underline{N}(x_i)^T \underline{\theta}}}{1 + e^{\underline{N}(x_i)^T \underline{\theta}}}}_{p(x_i)} \cdot \underbrace{\frac{1}{1 + e^{\underline{N}(x_i)^T \underline{\theta}}} \cdot \underline{N}(x_i) \underline{N}(x_i)^T}_{1 - p(x_i)}}_{\underline{N}(x_i) \underline{N}(x_i)^T} - \lambda \underline{\Omega} \\ &= - \sum_{i=1}^N p(x_i) (1 - p(x_i)) \underline{N}(x_i) \underline{N}(x_i)^T - \lambda \underline{\Omega} \\ &= - \underline{N}^T \underline{W} \underline{N} - \lambda \underline{\Omega}, \end{aligned}$$

where $\underline{W}$ is a diagonal matrix with diagonal entries $p(x_i)(1 - p(x_i))$