

Let $V_1, \dots, V_B$ be identically distributed samples
with $\text{Var}(V_i) = \sigma^2$, $i = 1, \dots, B$, and $\text{Corr}(V_i, V_j) = \rho$, $i \neq j$.

We have:

$$\text{Corr}(V_i, V_j) = \rho = \frac{\text{Cov}(V_i, V_j)}{\sqrt{\text{Var}(V_i) \text{Var}(V_j)}} = \frac{\text{Cov}(V_i, V_j)}{\sqrt{\sigma^2 \cdot \sigma^2}} = \frac{\text{Cov}(V_i, V_j)}{\sigma^2}$$

$$\longrightarrow \text{Cov}(V_i, V_j) = \sigma^2 \rho$$

We get:

$$\begin{aligned} \text{Var}\left(\frac{1}{B} \sum_{i=1}^B V_i\right) &= \frac{1}{B^2} \left(\sum_{i=1}^B \underbrace{\text{Var}(V_i)}_{=\sigma^2} + 2 \sum_{i=1}^{B-1} \sum_{j=i+1}^B \underbrace{\text{Cov}(V_i, V_j)}_{\substack{=\sigma^2 \rho}} \right) \\ &= \frac{1}{B^2} \left(B\sigma^2 + 2 \cdot \frac{B(B-1)}{2} \sigma^2 \rho \right) \\ &= \frac{1}{B} \sigma^2 + \sigma^2 \underbrace{\frac{B-1}{B}}_{1 - \frac{1}{B}} \\ &= \frac{1}{B} \sigma^2 (1 - \rho) + \rho \sigma^2 \end{aligned}$$