

We have:

$$R_0(\underline{\theta}) = \sum_{i=1}^N (y_i - f(\underline{x}_i))^2 + \lambda \left(\sum_{n=1}^M \sum_{j=1}^p \alpha_{jn}^2 + \sum_{n=1}^M \beta_n^2 \right)$$

with $f(\underline{x}_i) = g \left(\beta_0 + \sum_{n=1}^M \beta_n \sigma \left(\sum_{j=1}^p \alpha_{jn} x_{ij} \right) \right)$

$$\rightarrow \frac{\partial R_0(\underline{\theta})}{\partial \beta_n^*} = -2 \sum_{i=1}^N (y_i - f(\underline{x}_i)) \cdot \frac{\partial f(\underline{x}_i)}{\partial \beta_n^*} + 2\lambda \beta_n^*$$

$$\begin{aligned} \text{with } \frac{\partial f(\underline{x}_i)}{\partial \beta_n^*} &= g' \left(\underbrace{\beta_0 + \sum_{n=1}^M \beta_n \sigma \left(\sum_{j=1}^p \alpha_{jn} x_{ij} \right)}_{T_i} \right) \cdot \sigma \left(\sum_{j=1}^p \alpha_{jn} x_{ij} \right) \\ &= g'(T_i) \cdot \underbrace{\sigma \left(\sum_{j=1}^p \alpha_{jn} x_{ij} \right)}_{z_{in}^*} = g'(T_i) \cdot \sigma(z_{in}^*) \end{aligned}$$

Further, we have

$$\frac{\partial R_0(\underline{\theta})}{\partial \alpha_{j^*n^*}} = -2 \sum_{i=1}^N (y_i - f(\underline{x}_i)) \cdot \frac{\partial f(\underline{x}_i)}{\partial \alpha_{j^*n^*}} + 2\lambda \alpha_{j^*n^*}$$

with

$$\begin{aligned} \frac{\partial f(\underline{x}_i)}{\partial \alpha_{j^*n^*}} &= g' \left(\underbrace{\beta_0 + \sum_{n=1}^M \beta_n \sigma \left(\sum_{j=1}^p \alpha_{jn} x_{ij} \right)}_{T_i} \right) \cdot \beta_{n^*} \cdot \sigma' \left(\sum_{j=1}^p \alpha_{jn^*} x_{ij} \right) \cdot x_{ij^*} \\ &= g'(T_i) \cdot \beta_{n^*} \sigma' \left(\sum_{j=1}^p \alpha_{jn^*} x_{ij} \right) \cdot x_{ij^*} \end{aligned}$$