

We have:

$$\hat{y} = \operatorname{argmax}_k P(Y=k | \underline{X} = \underline{x})$$

$$= \operatorname{argmax}_k \left(\frac{\pi_k p_k(\underline{x})}{\sum_{l=1}^K \pi_l p_l(\underline{x})} \right)$$

$$= \operatorname{argmax}_k \left(\pi_k p_k(\underline{x}) \right)$$

$$= \operatorname{argmax}_k \left(\pi_k \cdot \frac{1}{(2\pi)^{p/2} |\underline{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\underline{x} - \underline{\mu}_k)^T \underline{\Sigma}^{-1} (\underline{x} - \underline{\mu}_k)} \right)$$

$$= \operatorname{argmax}_k \left(\log(\pi_k) - \frac{p}{2} \log(2\pi) - \frac{1}{2} \log |\underline{\Sigma}| - \frac{1}{2} (\underline{x} - \underline{\mu}_k)^T \underline{\Sigma}^{-1} (\underline{x} - \underline{\mu}_k) \right)$$

$$= \operatorname{argmax}_k \left(\log(\pi_k) - \frac{1}{2} \underline{x}^T \underline{\Sigma}^{-1} \underline{x} + \underline{\mu}_k^T \underline{\Sigma}^{-1} \underline{x} - \frac{1}{2} \underline{\mu}_k^T \underline{\Sigma}^{-1} \underline{\mu}_k \right)$$

$$= \operatorname{argmax}_k \left(\underbrace{\underline{\mu}_k^T \underline{\Sigma}^{-1} \underline{x} + \log(\pi_k) - \frac{1}{2} \underline{\mu}_k^T \underline{\Sigma}^{-1} \underline{\mu}_k}_{\mathcal{D}_k(\underline{x})} \right)$$

$$= \operatorname{argmax}_k \mathcal{D}_k(\underline{x})$$

We have:

$$\delta_u(\underline{x}) = \delta_l(\underline{x})$$

$$\longrightarrow \underline{\mu}_u^T \underline{\Sigma}^{-1} \underline{x} + \log(\pi_u) - \frac{1}{2} \underline{\mu}_u^T \underline{\Sigma}^{-1} \underline{\mu}_u = \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{x} + \log(\pi_l) - \frac{1}{2} \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{\mu}_l$$

$$\longrightarrow \underline{\mu}_u^T \underline{\Sigma}^{-1} \underline{x} - \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{x} = \frac{1}{2} \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{\mu}_u - \frac{1}{2} \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{\mu}_l - \log(\pi_u) + \log(\pi_l)$$

$$\longrightarrow (\underline{\mu}_u^T \underline{\Sigma}^{-1} - \underline{\mu}_l^T \underline{\Sigma}^{-1}) \underline{x} = \frac{1}{2} \underline{\mu}_u^T \underline{\Sigma}^{-1} \underline{\mu}_u - \frac{1}{2} \underline{\mu}_l^T \underline{\Sigma}^{-1} \underline{\mu}_l - \log(\pi_u) + \log(\pi_l)$$