Modeling by Stochastic Processes (STK 2130)
Exercises 1

Problem 1 Use the conditions (i), (ii) of the definition of a probability (see Def. 2.2 of the manuscript or appendix of Norris) to verify the following:

(i) $P = \delta_\omega$ given by

$$\delta_\omega(A) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{else} \end{cases} \quad \text{for events } A$$

is a probability.

(ii) If P is a probability then

$$P(A \cup B) \leq P(A) + P(B)$$

for all events A, B.

Problem 2 Compute the expected value of a random variable X, if

(i) X is Poisson distributed with parameter $\lambda > 0$, that is

$$P(X = n) = \frac{\lambda^n}{n!} e^{-\lambda}, n = 0, 1, 2, ...$$

(ii) X is exponentially distributed with parameter $\lambda > 0$, that is the density of its probability distribution is given by

$$f(x) = \lambda e^{-\lambda x}, x \geq 0.$$

Problem 3 Consider an equity-linked insurance policy given by a *Guaranteed Minimum Maturity Benefit* (GMMB), that is the policyholder (forsikringstaker) gets the payoff

$$\max(G, F_T),$$

where G is a guaranteed minimum return and F_T the value of a fund at terminal time T. Then the insurer's liability (forpliktelse) is given by the *put option*

$$\max(G - F_T, 0)$$

with strike price G.

Calculate the expected insurer's liability, that is

$$E\left[\max(G - F_T, 0)\right],$$

when $G = 4$ (million \$) and F_T at time $T = 1$ (year) is modelled by a Poisson random variable with parameter $\lambda = 7$.