Modeling by Stochastic Processes (STK 2130)
Exercises 2

Problem 1 The weather at a coastal resort is classified each day simply as "sunny" or "rainy". A sunny day is followed by another sunny day with probability 0.9, and a rainy day is followed by another rainy day with probability 0.3.

(i) Describe this as a Markov chain.

(ii) If Friday is sunny, what is the probability that Sunday is also sunny ?

(iii) If Friday is sunny, what is the probability that both Saturday and Sunday are sunny ?

Problem 2 Let S_n be the price of a stock at day $n = 0, 1, 2, 3$ ($n = 0$ present day). Suppose that $S_n \in \{0\$, 10\$, 20\$\}$ for all n and that the current stock price is 10\$, i.e. $S_0 = 10\$$. Further assume that stock prices $(S_n)_{0 \leq n \leq 3}$ are modelled by a Markov chain with transition matrix P given by

$$\begin{aligned} P &= \begin{pmatrix} p_{0\$0\$} & p_{0\$10\$} & p_{0\$20\$} \\ p_{10\$0\$} & p_{10\$10\$} & p_{10\$20\$} \\ p_{20\$0\$} & p_{20\$10\$} & p_{20\$20\$} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.6 & 0.35 \end{pmatrix}. \end{aligned}$$

What is the probability that the stock price reaches 20\$ at least on one of the the consecutive days $n = 1, 2, 3$?

Problem 3 Consider a sequence $(X_n)_{n \geq 0}$ of independent and identically distributed random variables with countable state space I such that

$$P(X_n = i) > 0$$

for all $n \geq 0, i \in I$.

Show that $(X_n)_{n \geq 0}$ is a Markov chain.

Problem 4 Let $(X_n)_{n \geq 0}$ be as in Problem 3. Define the process

$$S_n = X_0 + X_1 + ... + X_n,\ n \geq 0.$$

Is $(S_n)_{n \geq 0}$ a Markov chain ?

Problem 5 Let $B_1, B_2, ...$ be disjoint events with $\bigcup_{n=1}^{\infty} B_n = \Omega$. Show that if A is another event and $P(A|\ B_n) = p$ for all n then $P(A) = p$.

Deduce that if X and Y are discrete random variables then the following are equivalent:

(i) X and Y are independent.

(ii) The conditional probability of $X = i$ given $Y = j$ is independent of j for all i.