# Modeling by Stochastic Processes (STK 2130)
# Exercises 4

**Problem 1** Let $S_n$ be the price of a stock at time $n$ such that

$$S_n = 0\$ \text{ or } 1\$ \text{ or } 2\$, \ n = 0, 1, 2, ...$$

Assume that the stock prices $(S_n)_{n\geq 0}$ are modelled by a Markov chain with transition matrix $P$ given by

$$P = \begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.6 & 0.35 \end{pmatrix}.$$

Compute the probability that a stock trader with initial wealth $i$ $(i = 0, 1, 2)$ ever gets ruined.

**Problem 2** Consider a stock price process $S_n$ with values

$$S_n = 0 \ \$ \text{ or } 1 \ \$ \text{ or } 2 \ \$$$

for all $n = 0, 1, 2, ...$(in weeks).

Suppose that $(S_n)_{n\geq 0}$ is a Markov chain with transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0.15 & 0.5 & 0.35 \\ 0.07 & 0.68 & 0.25 \end{pmatrix}.$$

(i) Compute for all $i = 0, 1, 2$ the mean time for the stock price to hit 0 \$ given that its initial value is $i$ \$.

(ii) What is the mean time for the stock price to assume the values 0 \$ or 2 \$, given that $S_0 = 1$ \$, that is what is

$$E\left[H^A \,|S_0 = 1\right] \ ?$$

Here $H^A$ is the hitting time which is defined as

$$H^A = \inf\{n \geq 0 : S_n \in A\},$$

where $A = \{0, 2\}$.

(iii) A trader on a market, who observes the stock prices $S_n, n = 0, 1, 2, ...$, wants to buy the stock, if it costs 1 \$.

What is the probability that the stock price reaches the value 2 \$ three weeks after the purchase, that is what is

$$P\left(S_{T+3} = 2 \,|T < \infty\right) \ ?$$

Here $T = H^A$, where $A = \{1\}$.