Modeling by Stochastic Processes (STK 2130)
Exercises 5

Problem 1 Let S_n be the price of a stock at time n such that

$$S_n = 0\$ \text{ or } 1\$ \text{ or } 2\$, \ n = 0, 1, 2, ...$$

Assume that the stock prices $(S_n)_{n\geq 0}$ are modelled by a Markov chain with transition matrix P given by

$$P = \begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.6 & 0.35 \end{pmatrix}.$$

Calculate the mean time that a stock trader with initial wealth i $(i = 0, 1, 2)$ ever gets ruined.

Problem 2 Consider a stock price process S_n with

$$S_n = 0\$ \text{ or } 1\$ \text{ or } 2\$ \text{ or } 3\$$$

for all $n = 0, 1, 2, ...$ (in weeks).

Suppose that $(S_n)_{n\geq 0}$ is a Markov chain with transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.4 & 0.1 \\ 0.05 & 0.5 & 0.35 & 0.1 \\ 0.01 & 0.6 & 0.3 & 0.09 \end{pmatrix}.$$

A trader on the market who observes the dynamics of $S_n, n = 0, 1, 2, ...$ wants to buy the stock, if its price is at least 2\$.

(i) What is the probability that the trader's stock reaches 3\$ after two weeks given that its purchase price is 2\$? Or more precisely: Find the probability

$$P(S_{T+2} = 3 \,|T < \infty, S_T = 2),$$

where T is the hitting time

$$H^A = \inf\{n \geq 0 : S_n \in A\}$$

with $A = \{2, 3\}$.

(ii) Determine the corresponding probability for three weeks.

Problem 3 Consider the car insurance of Problem 1, Exercises 3. If you start as low-risk customer, how many years can you expect to stay there ?

Problem 4 Is the zero state of the stock price S_n in Problem 1 recurrent or transient ?