Modeling by Stochastic Processes (STK 2130)
Exercises 9

Problem 1 For which values of $0 \leq p \leq 1$ is the following matrix the transition matrix of a reversible Markov chain ?

$$\begin{pmatrix} 0 & p & 1-p \\ 1-p & 0 & p \\ p & 1-p & 0 \end{pmatrix}.$$

See Section 3.7 in the manuscript

Problem 2 Consider a Markov chain with state space $I = \{0, 1, 2, 3, 4, 5\}$ and transition matrix P given by

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(i) Describe the Markov chain by a diagram.

(ii) What is the smallest number of steps (excluding the zeroth step) in which a state can reach itself ?

(iii) What is the period of the chain ?

Problem 3 The transition matrix for a three-state Markov chain is

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

Show that the period of the chain is 2.

Problem 4 Consider a version of the success run chain of Exercises 7, Problem 3 where we disregard sequences of consecutive tails, in the sense that for example T, TT, TTT, and so on, all simply count as T.

(i) Describe this as a Markov chain and examine it in terms of irreducibility, recurrence and periodicity.

(ii) Find the invariant distribution and compare it with Exercises 7, Problem 3. Is it the limit distribution ?

See Section 3.6 in the manuscript

Problem 5 Have a look again at Problem 2. Find the invariant distribution. Is it the limit distribution ?