Problem 1

La $S_n \in \{0\$, 1\$, 2\$\}$, for $n = 0, 1, \ldots$ med overgangsmatrise

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.6 & 0.35 \end{pmatrix}$$

Definerer $h_i = P_i(\inf\{n \geq 0 : X_n = 1\})$.

Har $h_1 = 1$, $h_2 = 0.1 h_1 + 0.6 h_2 + 0.3 h_3$ og $h_3 = 0.05 h_1 + 0.6 h_2 + 0.35 h_3$ $\Rightarrow$ $h_2 = 0.1 + 0.6 h_2 + 0.3 h_3 \Rightarrow h_2(1 - 0.6) = 0.1 + 0.3 h_3 \Rightarrow h_2 = \frac{0.1}{0.4} + \frac{0.3}{0.4} h_3 = 0.25 + 0.75 h_3$

$\Rightarrow h_3 = 0.05 + 0.6(0.25 + 0.75 h_3) + 0.35 h_3 = 0.05 + 0.15 + (0.45 + 0.35) h_3 = 0.20 + 0.8 h_3 \Rightarrow h_3(1 - 0.8) = 0.2$

$\Rightarrow h_3 = 1 \Rightarrow h_2 = 1 \Rightarrow h_1 = 1.$