# Modellering av stokastiske prosesser (STK 2130)
# Løsningsforslag Exercises 4, Prob. 2

**Problem 2**

(i) By Theorem 3.2.6 in the manuscript (or Th. 1.3.5 in Norris) the mean hitting times $k_i^0, i = 0, ..., 3$ are the minimal non-negative solution of

$$\begin{cases} k_i^0 = 0, & \text{if } i = 0 \\ k_i^0 = 1 + \sum_{j=1}^{2} p_{ij} k_j^0, & \text{if } i \neq 0 \end{cases}.$$

This implies that

$$\begin{cases} k_0^0 = 0 \\ k_1^0 = 1 + 0.5k_1^0 + 0.35k_2^0 \\ k_2^0 = 1 + 0.68k_1^0 + 0.25k_2^0 \end{cases}.$$

So using substitution gives the unique solution

$$k_0^0 = 0, k_1^0 = 8.029, k_2^0 = 8.613 \text{ in weeks.}$$

(ii) The mean hitting times $k_i^A, i = 0, ..., 3$ are the minimal (possibly infinite) solution of

$$\begin{cases} k_i^A = 0, & \text{if } i \in A = \{0, 2\} \\ k_i^A = 1 + \sum_{j \notin A} p_{ij} k_j^A, & \text{if } i \notin A \end{cases}.$$

The latter equation becomes

$$\begin{cases} k_i^A = 0 \text{ for } i = 0, 2 \\ k_1^A = 1 + 0.5k_1^A \end{cases}.$$

So it follows $k_1^A \stackrel{def}{=} E[H^A \,|S_0 = 1] = 2$ weeks.

(iii) Because of the strong Markov property we have that

$$P(S_{T+3} = 2 \,|T < \infty \text{ and } S_T = 1) = P(S_3 = 2 \,|S_0 = 1) = p_{12}^{(3)}.$$

On the other hand the definition of $T$ implies that

$$S_T = 1 \text{ with probability 1.}$$

So we get

$$\begin{aligned} P(S_{T+3} &= 2 \,|T < \infty) = P(S_{T+3} = 2 \,|T < \infty \text{ and } S_T = 1) \\ &= P(S_3 = 2 \,|S_0 = 1) = p_{12}^{(3)}. \end{aligned}$$

Since

$$\begin{aligned} P^3 &= PPP \stackrel{def}{=} (p_{ij}^{(3)})_{0 \leq i,j \leq 2} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0.341 & 0.423 & 0.236 \\ 0.287 & 0.459 & 0.254 \end{pmatrix} \end{aligned}$$

we obtain

$$P(S_{T+3} = 2 \,|T < \infty) = p_{12}^{(3)} = 0.236.$$