

12.11.2015 First some comments on assignments, then ex 25, ex 26, p.2 exam 2013

from l) and f) you have shown.

$$\frac{SSE}{\sigma^2} \sim \chi^2_{I(J-1)} \quad \chi^2_{\frac{SST_r}{J\sigma_A^2 + \sigma^2}} \sim \chi^2_{I-1}$$

$$SSE + SST_r = SSR = \sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \bar{Y}_{..})^2 =$$

$$= \sum_{i=1}^I \sum_{j=1}^J \left| (Y_{ij} - \mu) - (\bar{Y}_{..} - \mu) \right|^2 =$$

$$= \sum_{i=1}^I \sum_{j=1}^J (Y_{ij} - \mu)^2 - 2(\bar{Y}_{..} - \mu) \sum_{i,j} (Y_{ij} - \mu) +$$

$$+ \sum_{i=1}^I \sum_{j=1}^J (\bar{Y}_{..} - \mu)^2 = \sum_{i,j} |Y_{ij} - \mu|^2 - IJ(\bar{Y}_{..} - \mu)^2$$

$$\stackrel{d}{=} (\sigma^2 + \sigma_A^2) \chi^2_{IJ} - \left( \frac{\sigma_A^2}{J} + \frac{\sigma^2}{IJ} \right) IJ (\bar{Y}_{..} - \mu)^2 \stackrel{d}{=} a_3^* \chi^2_{IJ} - a_4^* \chi^2_1 = \left( = \right) \left( \frac{\sigma_A^2 + \sigma^2}{IJ} \right)$$

$$= \underbrace{\chi^2_1 - \chi^2_2}_{a_1 \chi^2_1 - a_2 \chi^2_2} \Rightarrow \exists a_1, a_2 \neq 0$$

$$a_1 \chi^2_1 - a_2 \chi^2_2 \stackrel{d}{=} \chi^2_{IJ-1} \text{ - always true case}$$

$$SSE + SST_r = a_3^* \chi^2_{IJ(IJ-1)} + a_4^* \chi^2_{(I-1)} \Rightarrow$$

$$\Rightarrow \exists a_3, a_4 \neq 0$$

$a_3 SSE + a_4 SST_r = \left[ \text{! i. i. f. } \begin{matrix} SSE \perp SST_r \\ \text{independence} \end{matrix} \right]$

$= \chi^2_{(IJ-1) + (I-1)} = \chi^2_{IJ-1}$

!!! Linear transformation doesn't influence independence

26)  $\Sigma_{b_1} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1g} \\ & \ddots & & \vdots \\ & & \text{Symmetry} & \vdots \\ & & & \sigma_g^2 \end{pmatrix}$  - has  $g + \frac{g \times g}{2} = 45$  - parameters

$\Sigma_{b_2} = \begin{pmatrix} \sigma^2 & \sigma_{12} & \dots & \sigma_{1g} \\ & \ddots & & \vdots \\ & & \text{Symmetry} & \vdots \\ & & & \sigma_g^2 \end{pmatrix}$  - has  $1 + \frac{g \times g}{2} = 37$  parameters.

$\Sigma_{b_1, \text{wrong}} = \begin{pmatrix} \sigma^2 & & & \\ & \ddots & & \\ & & 0 & \\ & & & \ddots & \\ & & & & 0 & \\ & & & & & \ddots & \\ & & & & & & \sigma^2 \end{pmatrix}$  - has 1 parameter

