# UNIVERSITY OF OSLO

## Faculty of mathematics and natural sciences

Exam in: STK3405/STK4405 — Elementary introduction to risk and reliability analysis.

Day of examination: Wednesday 14. December 2016.

Examination hours: 15.00 – 19.00.

This problem set consists of 3 pages.

Appendices: None.

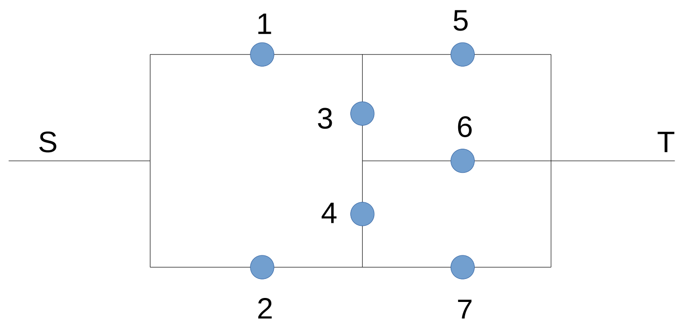
Permitted aids: Accepted calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

***All 13 subpoints will be equally weighted in the marking.***

## Problem 1

Consider the following flow network of independent component states.




a) Find the minimal path and cut sets of the system.

b) How many terms will we get in the best case (before we collect terms) by using the multiplication method to find the reliability of the system? Give a reason for your answer. Compare with the method based on total state enumeration (naming all of the states).

c) Find the reliability of this system as a function of the component reliabilities $p_1, \ldots, p_7$ using the factoring algorithm.

d) What is reliability importance of the 3. component according to the Birnbaum measure?

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e) What is the corresponding structural importance of the 3. component? Show this in two ways.

## Problem 2

Consider a monotone structure $\phi$ of $n$ components which are not repaired, and where the state processes of the components $\{X_i(t), t \geq 0\}_{i=1,\dots,n}$ are independent. Birnbaum (1969) suggested the following measure for the reliability importance of the $i$'th component at time $t$:

$$I_B^{(i)}(t) = \frac{\partial h(\boldsymbol{p}(t))}{\partial p_i(t)}, i = 1, \dots, n$$

a) Prove that:

$$I_B^{(i)}(t) = P[(1_i, \boldsymbol{X}(t)) \text{ is a critical path vector for component } i]$$

b) The Barlow and Proschan (1975) measure is defined by

$$I_{B-P}^{(i)} = P[\text{the } i\text{'th component causes directly that the system fails}], i = 1, \dots, n.$$

Show that:

$$I_{B-P}^{(i)} = \int_0^\infty I_B^{(i)}(t) f_i(t) dt,$$

where $f_i(t)$ is the probability density of the lifetime of the $i$th component.

c) The Vesely and Fussel (1975) measure is given by:

$$I_{V-F}^{(i)}(t) = P[X_i(t) = 0 | \phi(X(t)) = 0]$$

What arguments can be made against the use of this measure?

d) Assume that the $i$'th component is irrelevant for the structure $\phi$. In this case, what is $I_B^{(i)}(t), I_{B-P}^{(i)}$ and $I_{V-F}^{(i)}(t)$? Justify your answers and comment on them.

## Problem 3

a) Assume that $T_1, \dots, T_n$ are associated random variables such that $0 \leq T_i \leq 1$, $i = 1, \dots, n$. Show that:

$$E[\textstyle\prod_{i=1}^n T_i] \geq \textstyle\prod_{i=1}^n E[T_i]$$
$$E[\textstyle\coprod_{i=1}^n T_i] \leq \textstyle\coprod_{i=1}^n E[T_i].$$

*(Continued on page 3.)*

b) Consider a monotone structure $\phi$ with minimal path sets (cut sets) $P_1, \ldots, P_p$ $(K_1, \ldots, K_k)$. Prove that:

$$\max_{1 \leq j \leq p} P[\min_{i \in P_j} X_i = 1] \leq h \leq \min_{1 \leq j \leq k} P[\max_{i \in K_j} X_i = 1]$$

where $h = P[\phi(\boldsymbol{X}) = 1]$ is the system reliability.

c) Make the same assumptions as in b), but in addition assume that the component states are associated with component reliabilities $p_1, \ldots, p_n$. Show that in this case, the following holds:

$$\max_{1 \leq j \leq p} \prod_{i \in P_j} p_i \leq h.$$

d) Under the same assumptions as in c), prove that:

$$h \leq \min_{1 \leq j \leq k} \coprod_{i \in K_j} p_i$$

by applying the lower bound from c) on the dual structure $\phi^D$.

END