

UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: STK3405/4405 — Introduction to risk and reliability analysis

Day of examination: Monday December 12th 2022.

This problem set consists of 3 pages.

Appendices: None.

Permitted aids: Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

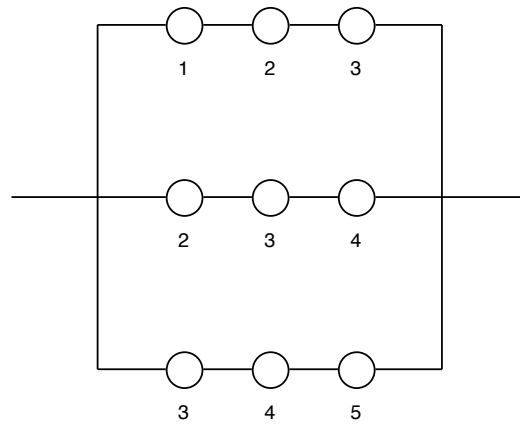


Figure 1: Block diagram a binary monotone system (C, ϕ)

Consider the binary monotone system (C, ϕ) shown in Figure 1. The component set of the system is $C = \{1, 2, 3, 4, 5\}$. Let $\mathbf{X} = (X_1, X_2, X_3, X_4, X_5)$ denote the vector of component state variables, and assume throughout this problem that X_1, X_2, X_3, X_4, X_5 are stochastically independent. Let $\mathbf{p} = (p_1, p_2, p_3, p_4, p_5)$ denote the vector of component reliabilities, where $p_i = P(X_i = 1)$, $i = 1, 2, 3, 4, 5$. We also introduce the *reliability function*:

$$h(\mathbf{p}) = P(\phi(\mathbf{X}) = 1) = E[\phi(\mathbf{X})]$$

(Continued on page 2.)

- a) Find the minimal path sets (3 sets) and the minimal cut sets (4 sets) of the system. Is the system *coherent*?

- b) Show that the structure function of the system is given by:

$$\phi(\mathbf{X}) = X_1X_2X_3 + X_2X_3X_4 + X_3X_4X_5 - X_1X_2X_3X_4 - X_2X_3X_4X_5$$

Compute $h(\mathbf{p})$ given that $\mathbf{p} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.

The Birnbaum measure of the *reliability importance* of component i is defined as:

$$I_B^{(i)} = P(\text{Component } i \text{ is critical for the system}), \quad i \in C.$$

- c) Show that:

$$I_B^{(i)} = \frac{\partial h(\mathbf{p})}{\partial p_i}, \quad i \in C.$$

- d) Use the result from (c) to show that:

$$\begin{aligned} I_B^{(1)} &= p_2p_3(1 - p_4) \\ I_B^{(2)} &= p_1p_3(1 - p_4) + p_3p_4(1 - p_5) \\ I_B^{(3)} &= p_1p_2(1 - p_4) + p_2p_4(1 - p_5) + p_4p_5 \end{aligned}$$

- e) Assume that $p_1 = p_2 = p_3 = p_4 = p_5 = p$, where $0 < p < 1$. Show that:

$$I_B^{(1)} = I_B^{(5)} < I_B^{(2)} = I_B^{(4)} < I_B^{(3)}$$

Problem 2

Let (C, ϕ) be a series system of non-repairable components where $C = \{1, \dots, n\}$, and let:

$$X_i(t) = \begin{cases} 1 & \text{if component } i \text{ is functioning at time } t \\ 0 & \text{otherwise} \end{cases}$$

for $t \geq 0$ and $i \in C$. Moreover, let $\mathbf{X}(t) = (X_1(t), \dots, X_n(t))$. We assume that the component processes $\{X_1(t)\}_{t \geq 0}, \dots, \{X_n(t)\}_{t \geq 0}$ are stochastically independent. The lifetime of component i is denoted by T_i , and we assume that the survival function of T_i is $\bar{F}_i$, for $i \in C$. Since the component processes are assumed to be stochastically independent, it follows that $T_1, \dots, T_n$ are stochastically independent as well, and we have:

$$P(T_i > t) = \bar{F}_i(t), \quad i \in C, \quad t \geq 0,$$

(Continued on page 3.)

a) Show that:

$$P(\phi(\mathbf{X}(t)) = 1) = \prod_{i=1}^n \bar{F}_i(t), \quad t \geq 0.$$

We then introduce the *time dependent* Birnbaum measure of *reliability importance* of component $i \in C$ at time $t \geq 0$ defined as:

$$I_B^{(i)}(t) = P(\text{Component } i \text{ is critical for the system at time } t)$$

b) Show that:

$$I_B^{(i)}(t) = \prod_{j \neq i} \bar{F}_j(t), \quad i \in C, \quad t \geq 0.$$

We then introduce the Barlow-Proshan measure of *reliability importance* of component $i \in C$ at time $t \geq 0$ defined as:

$$I_{B-P}^{(i)} = P(\text{Component } i \text{ fails at the same time as the system})$$

Assuming that $T_1, \dots, T_n$ are absolutely continuously distributed with densities $f_1, \dots, f_n$ respectively, it can be shown that we have:

$$I_{B-P}^{(i)} = \int_0^\infty I_B^{(i)}(t) \cdot f_i(t) dt$$

Finally, we assume that:

$$\bar{F}_i(t) = e^{-\lambda_i t^\alpha}, \quad i \in C, \quad t \geq 0.$$

where $\alpha > 0$ and $\lambda_i > 0$, for all $i \in C$.

c) Show that:

$$I_{B-P}^{(i)} = \frac{\lambda_i}{\sum_{j=1}^n \lambda_j}, \quad i \in C.$$

END