Monte Carlo methods

- Assume that you wish to assess a quantity ψ related to the distribution of a certain random variable X, for instance the standard deviation $\mathrm{sd}(X)$.
- Assume further that there is no analytical expression for $\mathrm{sd}(X)$.
- If you had independent observations $x_1, \ldots, x_m$ from the distribution in question, then you could estimate $\psi = \mathrm{sd}(X)$ by $\hat{\psi} = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (x_i - \bar{x})^2}$, where $\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i$.
- By the law of large numbers, the estimate $\hat{\psi}$ converges to the true value ψ.

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- The idea is to generate a synthetic data set from the given distribution by simulating from it.
- That is, for an appropriately large m:
  - draw m independent samples $x_1^*, \ldots, x_m^*$ from the given distribution
  - let $\hat{\psi}^* = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} \left(x_i^* - \bar{x}^*\right)^2}$.

Monte Carlo methods

- Another typical use of Monte Carlo simulation is for exploring how a process may evolve in the future.
- An application of this type is the *ruin problem*, that we will encounter later in the course.
- This consists in finding the start capital needed for the probability of negative net assets for the campany (ruin) to be smaller than a specified, very low level.
- Using Monte Carlo to solve this problem allows for much more complex and realistic models for the net assets than the classical mathematical solution.