

STK3505/4505 Mandatory assignment 1 of 1

September 28th, 2022

1 Submission deadline

Thursday 19th 2023, 14:30 in Canvas

2 Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with L^AT_EX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, your email address, course and assignment number. It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. ~~Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers.~~ All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you should include a code and a trial run, so that it is easy to see the result of the code. If you have problems attaching the code to the pdf-file, you can just include the screenshots of your code and the trial run.

3 Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination. I cannot postpone the deadline.

4 Complete guidelines about delivery of mandatory assignments:

<https://www.uio.no/english/studies/examinations/compulsory-activities/mn-math-mandatory.html>

Good luck!!!

5 Assignments

5.1 Assignment 1 - Time value of money (30 p)

A perpetual bond (known also as a console) is a financial instrument characterised with coupon c (which is assumed to be paid out annually) and principal P . For such a bond, a bond holder gets cP amount of money at the end of each year for infinity. The perpetual bond does not have therefore any maturity time. The value of the bond at time $t = 0$ is equal to the present value of the underlying cashflows.

1. Assume that the interest rate at time $t = 0$ is flat and equal r . Prove that the value of a perpetual bond is given by $\frac{cP}{r}$.
2. Prove that the duration of a perpetual bond is given by $\frac{1+r}{r}$. Hint: Differentiate equation: $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$ w.r.t. q . You do not need to prove that you can go with the derivative within the infinite sum.
3. Write a function which calculates the value of the perpetual bond, given parameters r, c, P . Run the function for $r = 5\%$, $c = 4\%$ and $P = 1000$. Run the function again changing different parameters and plot the results (parameter against the value of the bond), first with the interest rate ($r = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$), then coupon ($c = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$) What does the results tell you about the relation between different level of parameters and the value of the bond?
4. Calculate the duration of the bond for $r = 5\%$, $c = 4\%$ and $P = 1000$ and give an interpretation to it. Assume that the interest rate has increased from $r = 5\%$ to $r = 6\%$. How much would the price of the bond change, as prescribed by the duration? What is the real change in the value of the bond? Why those two value are different?

5.2 Assignment 2 - Monte Carlo and parameter estimation (40 p)

We assume that a random sample $Z_1, \dots, Z_n$ is Pareto-distributed as defined in the lecture.

1. Derive an inverse sampler for a Pareto-distribution (Note: You are expected to compute the inverse of CDF).
2. Write a log-likelihood function $\mathcal{L}(\alpha, \beta)$. Derivate $\mathcal{L}(\alpha, \beta)$ w.r.t. α and find a critical point $\hat{\alpha}_\beta$, i.e. a value such that $\frac{\partial \mathcal{L}(\hat{\alpha}_\beta, \beta)}{\partial \alpha} = 0$. Argue that a maximum likelihood estimator is a point which maximizes the following function:

$$\beta \mapsto \mathcal{L}(\hat{\alpha}_\beta, \beta)$$

3. Write a function which calculates the maximum likelihood estimators given a sample $Z_1, \dots, Z_n$. Function should be general enough to handle different n . Hint: As you won't be able to find an estimator analytically, you need to use numerical methods to optimize the log-likelihood function (as argued in the previous point). Function *optimize* in R can be used to find a local minimum numerically.
4. Generate $n = 10, 100, 1000, 10000, 1000000$ simulations using the inverse sampler (simulations for different n should be drawn independently) for parameters $\alpha = 5$ and $\beta = 2$. Compute a maximum likelihood estimator $(\alpha_{ML}, \beta_{ML})$ for each n using a function from a previous point and prepare a table presenting the estimated parameters for different n . Compare it to the true value of parameters and comment on the estimation error.
5. Using the closed-form formula compute mean, standard deviation and 0,025 and 0,975-quantiles for **the Pareto-distribution with** estimated parameters $(\alpha_{ML}, \beta_{ML})$ for each n estimated in the previous point, as well as for true value of the parameters. Present the values in the table and comment on them.

5.3 Assignment 3 - Non-life insurance (20 p)

We assume the compound Poisson model for $J = 1000$ policies. In this model, the claim number from all policies is modelled as a random variable $N \sim Poiss(\lambda)$, ($\lambda = J\mu T$, $\mu = 0.1$, $T = 1$) and the claim size Z_i is distributed according to lognormal distribution with parameters $\mu = 2$, $\sigma = 3$. Then the sum of all claims can be defined as $X = \sum_{i=1}^N Z_i$.

1. Compute the expectation and the standard deviation of X using the closed-form formula proven on the lecture. What is a pure premium for one policy from that portfolio?
2. Write a program which simulates the sum of claims X and simulate $n = 10, 100, 1000, 10000$ simulations. Compute the mean and standard deviation from the simulation for different n and compare it with point 1.