

# **Introduction to insurance mathematics**

**Autumn 2024, UiO**

# Non-life insurance contract over time

I.e. how to set up reserves and update reserves  
over time



In curriculum but not in book

# Reserving in the book



In Chapter 1, section 1.2.3. reserves (or solvency capital) are defined as  $(1 - \epsilon)$ -quantile of a process  $X = \sum_{i=1}^N Z_i$ ,  $N$ -random number of claims,  $Z_i$  - size of  $i$ -th claim. Such a definition is **wrong** and **confusing** because:

- Does not take into account running of the time, reporting of the claims and payments
- solvency capital is a part of equity/own funds, not a part of reserves
- reserves often consists of a “best estimate”, i.e. expected amount, and “risk margin/adjustment” which measures the risk that the full amount paid out is different than the expectation. However neither risk margin nor risk adjustment is defined as percentile of  $X$ .

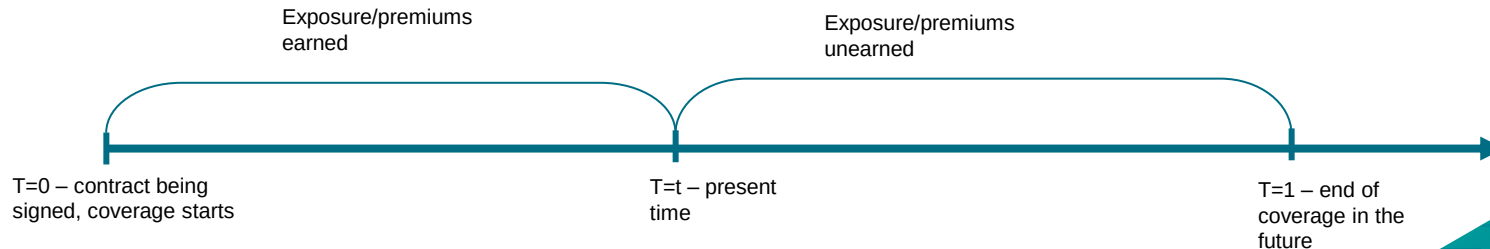
Therefore, I would like you to forget about this definition.

# A priori risk vs. real experience



When pricing, we use the model to analyse the risk during the course of coverage of an insurance product but BEFORE that coverage starts. This risk then is called **a priori** risk (*a priori*=“from the earlier” in Latin)

- However, after the contract is signed, and the coverage starts, we get some knowledge about the risk and we use this to set reserves.
- Firstly, we distinguish between **unearned premiums/exposure** and **earned premiums/exposure**
- Earned exposure/premiums** is associated with the part of coverage which has already happened, i.e. claims which have already occurred. Based on reported claims, we have more information about the what has happened and we can reflect that in our estimates
- Unearned exposure/premiums** is connected to the part of the coverage period in the future, i.e. future claims. Here we still do not know what is going to happen and a priori model is still what we have



# Calculation of earned/unearned/paid premium – policy level

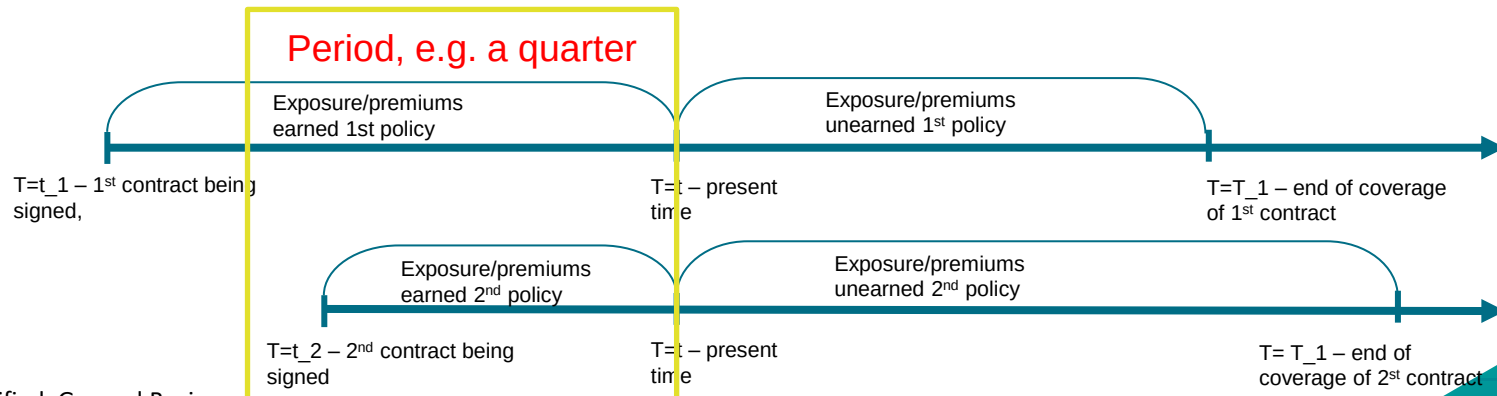


- In most cases the exposure is earned linearly – i.e. we assume that underlying risks are uniformly distributed w.r.t. time
- Then  $\text{earned premium} = \frac{\text{time since signing the contract}}{\text{the length of the coverage}} * \text{overall premium}$  and  $\text{unearned premium} = \text{overall premium} - \text{earned premium}$ .
- However in some cases better to use other models for earning the exposure (e.g. motorbike insurance in Norway)
- Note that earned premium is not the same as paid premium. Paid premiums reflects what has been paid to the insurance company, regardless if it has been earned or not.
- Different payment schedules:
  - Upfront one-time premium
  - Monthly paid premiums, quarterly premiums etc.

# Premiums per portfolio level – some definitions

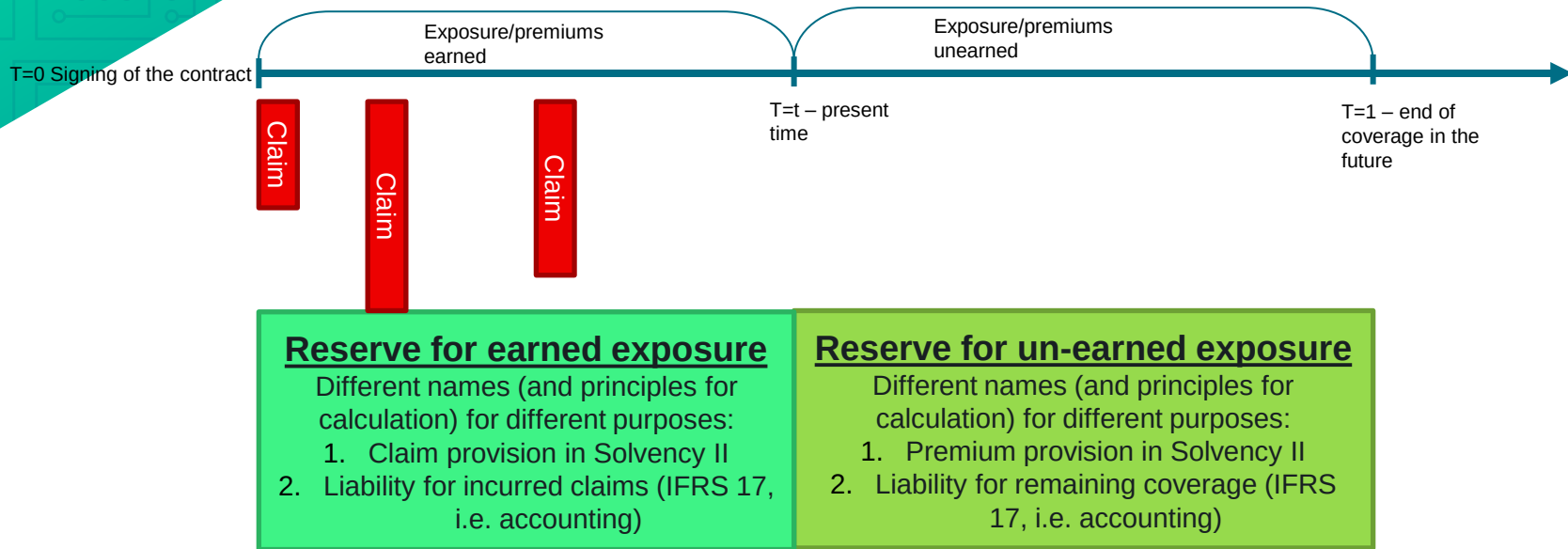


- **Written premiums** from a period – the sum of all the premium amounts (regardless if they been paid or not, earned or not) from contracts signed in a given period – measure of the size of the company
- **Earned premiums** from a period – the sum of all the premiums which have been earned in the period ( regardless if they have been paid or not, nor when the contracts have been signed) – measure for the risk exposure in that period
- Often one additionally adds “gross” or “net” to indicate if the reinsurance premiums have been deducted from the sum, e.g. net earned premiums.





# From earning exposure to reserving – policy level



Solvency II – EU regulation which defines e.g. how much capital insurance company must hold, how to calculate that capital, among other things how to calculate reserves

IFRS 17 – new accounting standard which is going to be implemented from January 1<sup>st</sup> 2023 for listed insurance companies

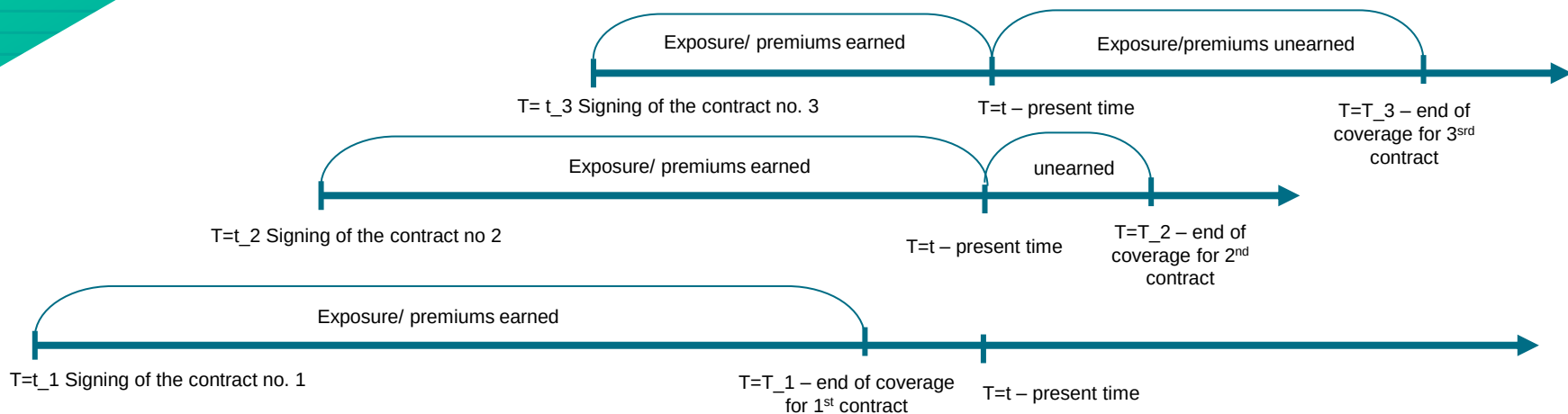


# Some notes

- ① A if a policy has just been signed, it will have only reserve for un-earned exposure
- ② If some time passed, there will be reserves for both unearned and earned exposure
- ③ After the end of the coverage, there is no reserve for un-earned exposure, but there still might be a reserve for earned exposure, as long as all the claims have not been settled.



# From earning exposure to reserving – Portfolio level



Reserve for earned exposure

Reserve for un-earned exposure

# Reserve for unearned exposure

## Solvency II

*(Best estimate of) Premium Provision* =  
discounted sum of expected claims and  
of expected expenses minus  
discounted sum of expected future  
premiums connected to unearned  
exposure from all the active policies

## IFRS17

*Liability for remaining coverage* =  
unearned premiums minus sum of  
expected future premiums connected  
to unearned exposure from all the  
active policies (minus acquisition costs i.e.  
commissions to the brokers etc.)

# Example – reserve for unearned exposure, quarterly premium



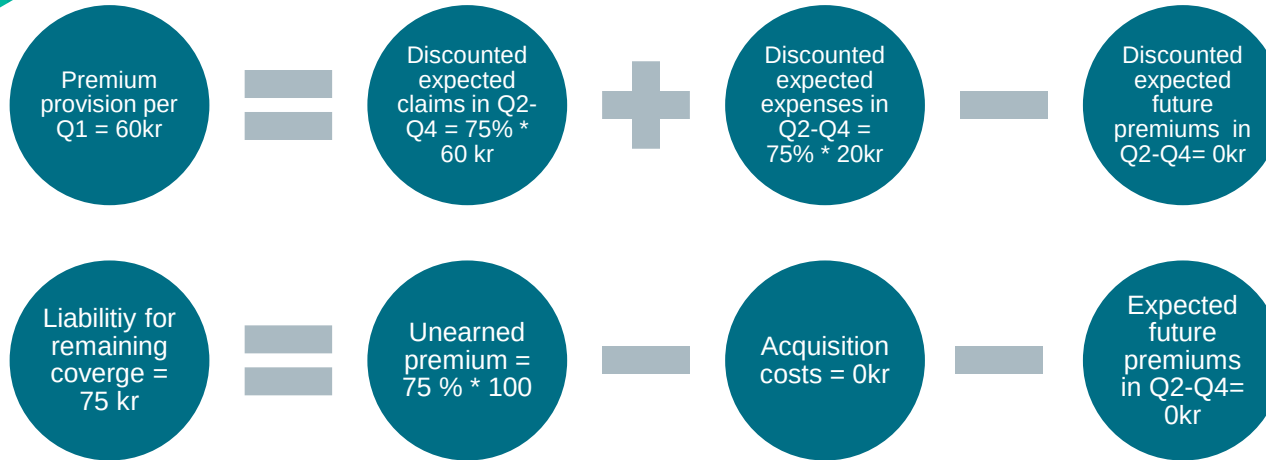
Assume that at January 1<sup>st</sup> we signed an insurance contract with premiums 100 kr, with premiums paid either:

1. Upfront for the whole year
2. Quarterly at the end of the quarter,
3. Quarterly, but upfront at the beginning of each quarter.

Assume that exposure is earned linearly and the insurer expects that it is going to pay as claims total 60 kr and as expenses 20 kr. Interest rate is 0% and acquisition expenses are 0.

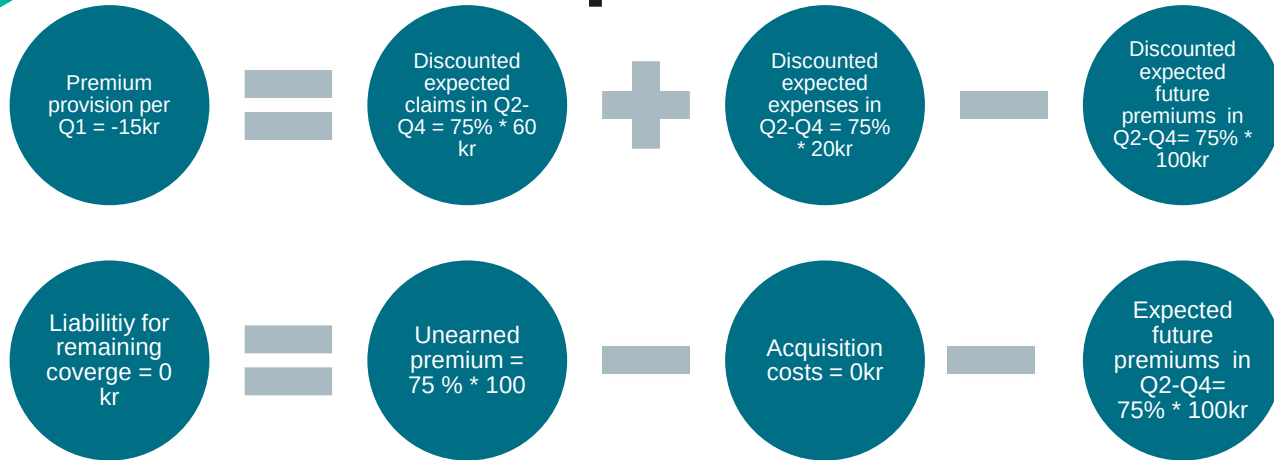
What is the reserve after a first quarter?

# Example – reserve for unearned exposure, upfront premium



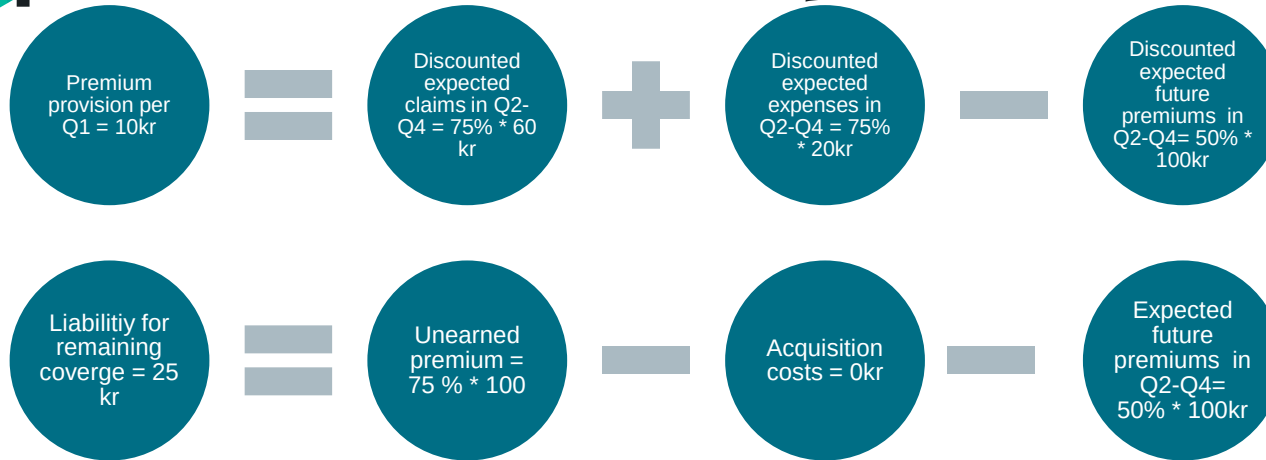
Difference is 15 kr – Can anyone give an interpretation to this amount?

# Example – reserve for unearned exposure, quarterly premium paid at the end of the quarter



The difference from Solvency II still is 15kr.

# Example – reserve for unearned exposure, quarterly premium paid upfront each of the quarter



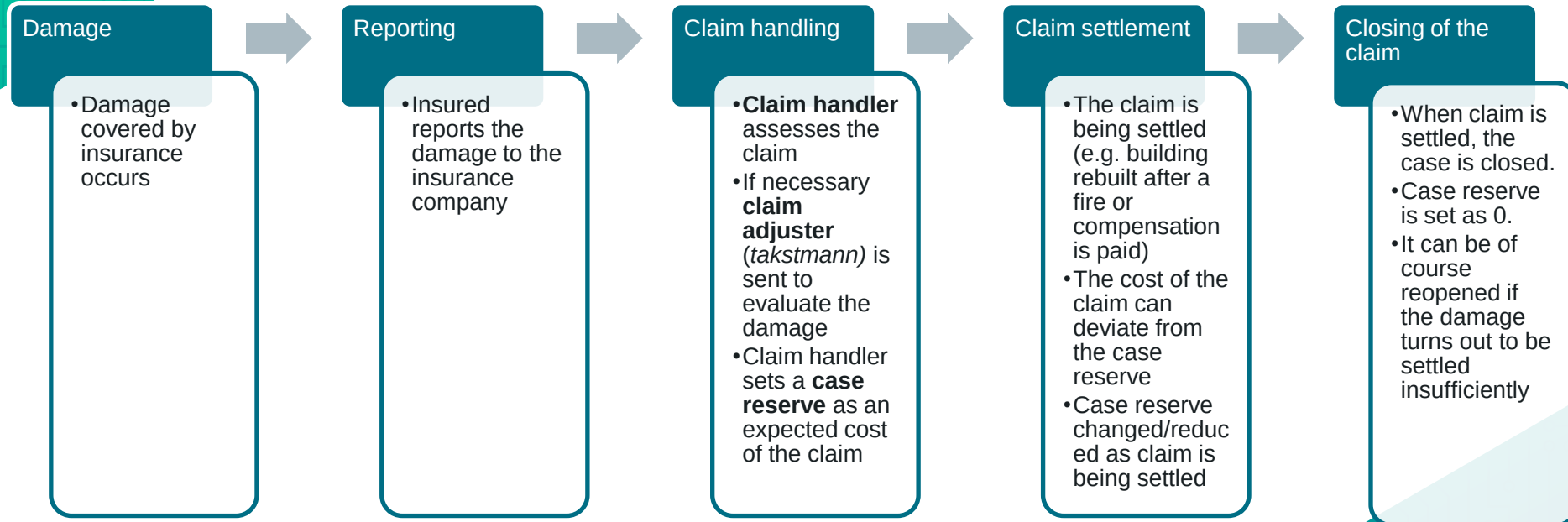
The difference from Solvency II still is 15kr.

# How our compound Poisson model is used for unearned exposure?



- ① IFRS17 – not used at all
- ② Solvency II – we can calculate expected claims using compound Poisson model, by just modifying  $T$  in the intensity of the Poisson distribution

# Earned exposure – process of claim handling and reserving



# Reserve for earned exposure

## Solvency II

*(Best estimate of) Claim Provision =*

discounted sum of expected claims and of expected expenses connected to earned exposure from all the policies, regardless when the policy was written

Solvency II has also an element of reserves called Risk Margin which serves the same purpose as risk adjustment, but is calculated for both unearned and earned exposure and with different method. However, we are not that interested here in risk margin/adjustment and we will focus just on Claim Provision

## IFRS17

*Liability for incurred claims =*

discounted sum of expected claims and of expected expenses connected to earned exposure from all the policies, regardless when the policy was written

PLUS

risk adjustment which measures the uncertainty of those amounts claims

# From claim handling to reserve for earned exposure



- **Total reserve** (undiscounted) = **RBNS** + **IBNR** + **Expenses**
- **RBNS** (reported but not settled) = sum of all the case reserves for open claims – comes from claim handling division
- **IBNR** (incurred but not reported) = the reserve calculated by an actuary for all the claims that happened but have not been yet reported and for claims that have been reported but the real cost can be larger than the case reserve (i.e. reported insufficiently)
- **Expenses** come e.g. from the budget



# How to model IBNR

Naïve approach – We fully believe in a priori estimate:

- Compound Poisson model says that we should expect  $X$  kr of claims, we have paid out  $Y$  kr and set  $Z$  kr as remaining case reserves/RBNS, therefore  $IBNR = X - Y - Z$
- Does not take into account the information in the case reserves or paid claims.
- Chain Ladder (CL): we analyse development pattern
  - at time  $t$  we usually pay out  $x\%$  of all the claims. We believe that the claims paid out so far give us information on the unreported claims.
  - so if we paid out  $Y$  so far, then  $IBNR = Y/x - Y - Z$ . More detailed definition comes later.
  - May give excessively volatile reserves, best for exposure where payment data or case reserves sufficiently reliable
- Bornhuetter-Ferguson (BF): slightly less naïve approach:
  - at time  $t$  we usually pay out  $x\%$  of all the claim, but we do not believe that it gives us any new information regarding the unreported claims and we still believe in a priori for those claims.
  - Then:  $IBNR = X \cdot (1 - x) - Z$
- Combination of both – Benktander
  - Weighting Chain Ladder estimates and BF and CL, giving BF more weight for newly earned exposure and CL estimate for older exposure

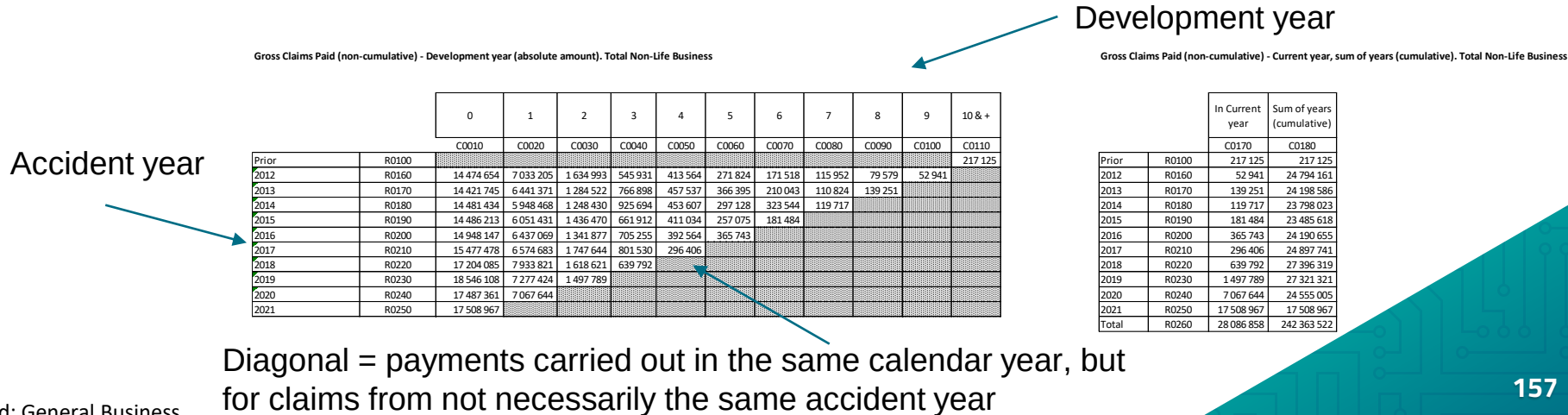
# How to estimate payment pattern?

## Claim triangle

Starting point – a claim triangle, which groups the payments attributed to claims in two dimensions: accident period (i.e. period when damage happened) and development period (i.e. period, measured from the accident date, when a particular payment was made)

E.g. gross claim triangle from If P&C Insurance from the regulatory reporting

(<https://www.sampo.com/globalassets/arkisto/hallintomateriaali/sfcr/2021/quantitative-reporting-templates-2021-if-pc-insurance-ltd..xlsx> Sheet 5.19.01.21)



# How to estimate payment pattern? – A claim triangle cont.

Start with a historical INCREMENTAL *paid* claim triangle

$$I = [I_{ij}]_{\substack{i - \text{accident period} \\ j - \text{development period}}} = \begin{bmatrix} I_{1,1} & I_{1,2} & \cdots & I_{1,n-1} & I_{1,n} \\ I_{2,1} & I_{2,2} & \cdots & I_{2,n-1} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ I_{n-1,1} & I_{n-1,2} & \cdots & NA & NA \\ I_{n,1} & NA & \cdots & NA & NA \end{bmatrix}, \text{ where } I_{ij} = \text{incremental amount of payments}$$

for i-th accident period and until the j-th development period either on paid

Lets build a historical CUMULATIVE *paid* claim triangle

$$C = [C_{ij}]_{\substack{i - \text{accident period} \\ j - \text{development period}}} = \begin{bmatrix} C_{1,1} & C_{1,2} & \cdots & C_{1,n-1} & C_{1,n} \\ C_{2,1} & C_{2,2} & \cdots & C_{2,n-1} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{n-1,1} & C_{n-1,2} & \cdots & NA & NA \\ C_{n,1} & NA & \cdots & NA & NA \end{bmatrix}, \text{ where } C_{ij} = \sum_{k=1}^j I_{i,k} - \text{i.e. we sum all the}$$

payments per accident year i until development year j

# How to compute payment pattern? – Chain Ladder

In previous slide  $C$  was historical and therefore there were “NA” entries. What we want to do is treat  $C$  as a matrix of random variables.

## Theorem 1 (Mack)

Let  $C = [C_{ij}]_{i,j=1,\dots,N'}$  be a cumulative claim triangle satisfying the following two conditions

1.  $(C_{i,1}, C_{i,2}, \dots, C_{i,N})$  and  $(C_{j,1}, C_{j,2}, \dots, C_{j,N})$  are independent for different  $i$  and  $j$
2.  $E[C_{i,k+1} | C_{i,1}, C_{i,2}, \dots, C_{i,k}] = C_{i,k} * f_k$  for some constants  $f_k$  called development factors.

Denote by  $D = (C_{ij})_{i,j \leq N, i+j \leq N+1}$  (observed triangle up to  $N$ ) Then

$$E[C_{i,N} | D] = C_{i,k} * f_k * \dots * f_{N-1}$$

# How to compute payment pattern? – Chain Ladder

In previous theorem, we assumed existence of some constants (development factors)  $f_k$  which were crucial to estimate the final amount. Now we would like to estimate that development factors.

## Theorem 2 (Mack)

Let  $C = [C_{ij}]_{i,j=1,\dots,N'}$  be a cumulative claim triangle satisfying the following two conditions

1.  $(C_{i,1}, C_{i,2}, \dots, C_{i,N})$  and  $(C_{j,1}, C_{j,2}, \dots, C_{j,N})$  are independent
2.  $E[C_{i,k+1} | C_{i,1}, C_{i,2}, \dots, C_{i,k}] = C_{i,k} * f_k$

Define  $\hat{f}_k$  (k-th Chain Ladder development factor) calculated as

$$\hat{f}_k = \frac{\sum_{i=1}^{N-k} C_{i,k+1}}{\sum_{i=1}^{N-k} C_{i,k}}.$$

*Measures how much on average the cumulative claim amounts increase from one development period to another*

Set  $B_k = (C_{ij})_{j \leq k, i+j \leq N+1}$  for  $k \leq N$ . Then

$E[\hat{f}_k | B_k] = f_k$ ,  $E[\hat{f}_k] = f_k$  and  $\hat{f}_k$  and  $\hat{f}_l$  are uncorrelated for different  $k$  and  $l$ . Moreover,

$$E[\hat{f}_{N-i+1} * \dots * \hat{f}_{N-1}] = E[\hat{f}_{N-i+1}] * \dots * E[\hat{f}_N]$$

# On theorems

$C_{i,N}$  - cumulative amount at the last development year – i.e. what we will pay out in total. This is also called an ULTIMATE.

$E[C_{i,N}|D]$  - estimate of the ultimate given the observed triangle.

Therefore our reserve for an accident year  $i$  given observed triangle  $D$  can be expressed as

$$R(i|D) = E[C_{i,N}|D] - C_{i,N-i+1} = \underbrace{C_{i,N-i+1} f_{N-i+1} * \dots * f_{N-1}}_{\text{Estimate of ultimate}} - \underbrace{C_{i,N-i+1}}_{\text{What we paid so far}}$$

Let's include our estimates via chain ladder development factors to get an unbiased estimator of reserves

$$\hat{R}(i|D) = \hat{C}_{i,N} - C_{i,N-i+1} = C_{i,N-i+1} * \hat{f}_{N-i+1} * \dots * \hat{f}_{N-1} - C_{i,N-i+1}$$

Since we used here a paid cumulative triangle, the reserve here is both the sum of RNBS and IBNR.

Assume however that we use an INCURRED triangle: consisting of cumulative paid amounts increased by RBNS at that period. If that incurred triangle is used for Chain Ladder, then our reserve calculated as above will give us the estimate of IBNR only.

# Proof: Theorem 1

Proof:

Introduce a notation:  $E_i[X] := E[X|C_{i,1}, \dots, C_{i,N-i+1}]$ .

We get that

$$E[C_{i,N}|D] = \text{by (1)} = E_i[C_{i,N}] = E_i[E[C_{i,N}|C_{i,1}, \dots, C_{i,N-1}]] = \text{by (2)} = E_i[C_{i,N-1}f_{N-1}] = E_i[C_{i,N-1}] f_{N-1} = \\ \text{repeat the argument} = E_i[C_{i,N-i+1}]f_{N-i+1} * \dots * f_{N-1} = C_{i,N-i+1}f_{N-i+1} * \dots * f_{N-1}.$$

# Proof: Theorem 2

Proof:

Fix  $k$ . We remember that  $B_k = (C_{ij})_{j \leq k, i+j \leq N+1}$ . Then:

$$E[C_{i,k+1}|B_k] = \text{by (1)} = E[C_{i,k+1}|C_{i,1}, \dots, C_{i,k}] = \text{by (2)} = C_{i,k} f_k. (*)$$

Then

$$E[\hat{f}_k|B_k] = E\left[\frac{\sum_{i=1}^{N-k} C_{i,k+1}}{\sum_{i=1}^{N-k} C_{i,k}} \middle| B_k\right] = \frac{E[\sum_{i=1}^{N-k} C_{i,k+1}|B_k]}{\sum_{i=1}^{N-k} C_{i,k}} = \frac{\sum_{i=1}^{N-k} E[C_{i,k+1}|B_k]}{\sum_{i=1}^{N-k} C_{i,k}} = \text{by } (*) = \frac{\sum_{i=1}^{N-k} C_{i,k} f_k}{\sum_{i=1}^{N-k} C_{i,k}} = f_k$$

Without the loss of generality assume that  $l < k$ . Then:

$$E[\hat{f}_k \hat{f}_l] = E[E[\hat{f}_k \hat{f}_l|B_k]] = E[\hat{f}_l * E[\hat{f}_k|B_k]] = E[\hat{f}_l * f_k] = E[\hat{f}_l] * f_k = E[\hat{f}_l] * E[\hat{f}_k].$$

Similarly:

$$\begin{aligned} E[\hat{f}_{N-i+1} * \dots * \hat{f}_{N-1}] &= E[E[\hat{f}_{N-i+1} * \dots * \hat{f}_{N-1}|B_{N-1}]] = E[\hat{f}_{N-i+1} * \dots * \hat{f}_{N-2} E[\hat{f}_{N-1}|B_{N-1}]] \\ &= E[\hat{f}_{N-i+1} * \dots * \hat{f}_{N-2}] * E[\hat{f}_N] = \text{same computations} = E[\hat{f}_{N-i+1}] * \dots * E[\hat{f}_N] \end{aligned}$$

# Chain Ladder – example (done during exercise classes)

- See the Excel sheet for the example for If P&C Insurance

# Payment pattern for a priori distribution and discounting of premium provision—outside curriculum

- Premium provision is discounted sum of expected claims and of expected expenses minus discounted sum of expected future premiums connected to unearned exposure from all the active policies
- So far we only considered undiscounted total amount. How to discount it?
- We know when we are to get premiums and pay expenses – no problem with discounting
- Only problem with when claims are to be paid => use payment pattern from Chain Ladder
- To be more exact: easy to check that if  $\hat{f}_j$  is j-th development factor,  $j=1,..,n$ , then the percentage paid in j-th year can be calculated as:

$$a_j = \frac{(\prod_{i=1}^{j-2} \hat{f}_i) * (\hat{f}_{j-1} - 1)}{\prod_{i=1}^{n-1} \hat{f}_i}$$

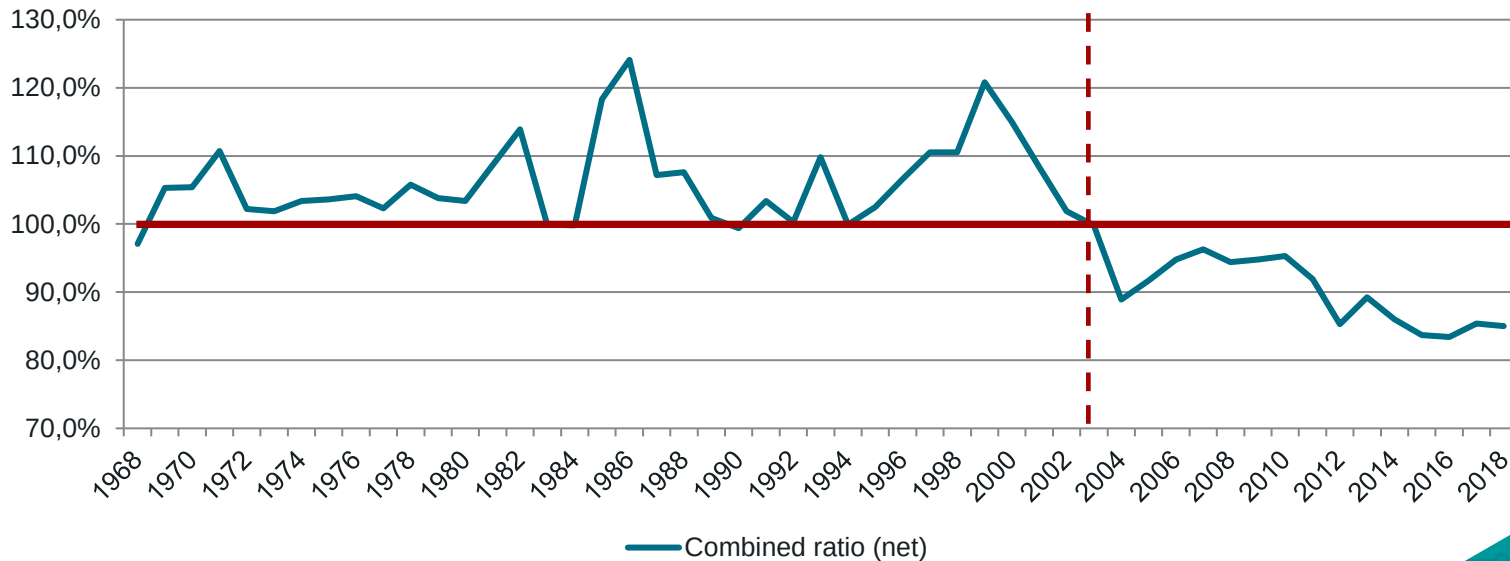
- Then we can just multiply the total amount of claims by  $a_j$  to get the estimate of the cashflow in year j-th. Having cashflows, we can discount them.

# Loss ratio and combined ratio

- Two very important ratios describing the performance and profitability of portfolio
- Loss ratio="incurred claims in the period" over "premiums earned in the period"
- $$\text{Loss ratio} = \frac{\text{claims paid out in the period} + \text{change in claim provision in the period}}{\text{earned premiums}}$$
- Combined ratio="incurred claims and expenses in the period" over "premiums earned in the period"
- $$\text{Combined ratio} = \frac{\text{claims paid out in the period} + \text{expenses in the period} + \text{change in claim provision in the period}}{\text{earned premiums}}$$
- Loss ratio in Scandinavia is ca. 70%. Combined ratio is ca. 85%
- If combined ratio is over 100% then the company is losing money on insurance activities

# Historical development of combined ratio for Gjensidige Forsikring

Combined ratio



# Pricing of an insurance contract in real life

Risk differentiation



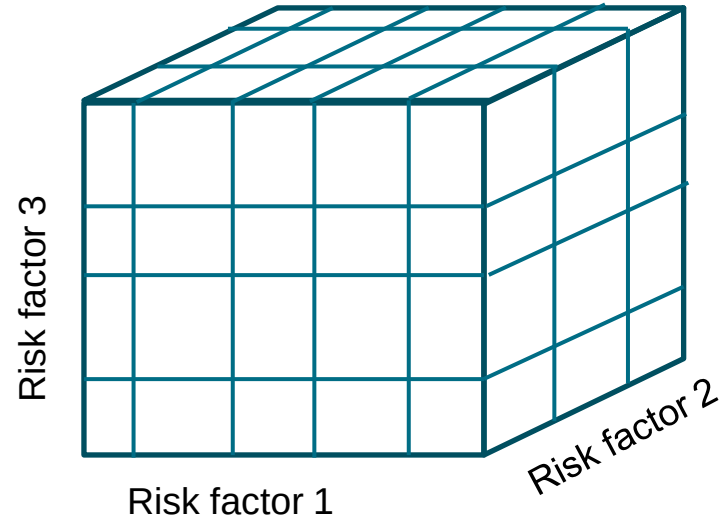
# Pricing so far

- ① We assumed a compound Poisson model, either on portfolio basis or on a policy basis, and we calculated pure premium based on it.
- ② For model per policy, we assumed that intensity of the Poisson process and the parameters of the claim severity distribution may depend on policy
- ③ In this section we wonder how we can come up with the parameters per policy

# Risk differentiation



- Assume that we suspect that some variables can determine the frequency of claims, e.g. a young driver tends to drive faster than an older one and therefore causes more accidents. We call such a variable a **risk factor**
- Risk differentiation** is a process to determine such risk factors and divide the portfolio into a small groups with similar risks. Those groups will get the same pure premium
- Risk differentiation can be done both for the claim frequency and claim size. We will focus mostly on modelling risk differentiation for claim frequency



# Motivation – exponential family of distributions (outside curriculum)

- Exponential family distribution contains the distributions (discrete and continuous) for which the probability function can be written as:

$$f(y) = c(y, \phi) \exp \left\{ \frac{y\theta - a(\theta)}{\phi} \right\}$$

- $\theta \in \mathbb{R}$  is called a canonical parameter telling something about the location,  $\phi$  is a dispersion parameter,  $a(\theta)$  and  $c(y, \phi)$  are function
- We know moreover that if  $X$  is distributed according to  $f$  then
  - $E(X) = \frac{d}{d\theta} a(\theta)$  and  $Var(X) = \phi \frac{d^2}{d\theta^2} a(\theta)$
- For Poisson we know that the probability function can be written as

$$p(k, \lambda) = \frac{\lambda^k}{k!} e^{-\lambda} = \frac{1}{k!} e^{k \ln \lambda} e^{-\lambda} = \frac{1}{k!} e^{k \ln \lambda - \lambda}$$

Comparing the terms we get that

$$c(y, \phi) = \frac{1}{y!}, \quad \theta = \ln \lambda, \quad a(\theta) = e^\theta, \quad \phi = 1$$

# Motivation – generalized linear models (outside the curriculum)

- We assume that our frequency depends on some risk factors  $X_1, \dots, X_n$
- E.g. for car insurance  $X_1, \dots, X_n$  can be an age of the driver, the power of the engine etc.
- In regular regression we want to model:

$$E(Y|X_1, \dots, X_n) = \alpha_0 + \alpha_1 X_1 + \dots + \alpha_n X_n$$

- However, regular regression can produce negative numbers, therefore its not a good idea if Y is Poisson
- However, we know that for Poisson

$$E(Y|X_1, \dots, X_n) = \frac{d}{d\theta} a(\theta_{|X_1, \dots, X_n}) = \exp \theta_{|X_1, \dots, X_n}$$

And  $\theta$  can take all real values and can be modelled therefore with a linear regressors  
Therefore we define our model as

$$E(Y|X_1, \dots, X_n) = \exp \theta_{|X_1, \dots, X_n} = \exp(\alpha_0 + \alpha_1 X_1 + \dots + \alpha_n X_n)$$

# Poisson regression. Why it is nice?

- ① Assume the model  $E(N|X_1, \dots, X_n) = \exp(\alpha_0 + \alpha_1 X_1 + \dots + \alpha_n X_n)$ ,  $N \sim \text{Poiss}(\lambda)$
- ①  $E(N|X_1, \dots, X_n) = \exp(\alpha_0 + \alpha_1 X_1 + \dots + \alpha_n X_n) = \exp(\alpha_0) \exp(\alpha_1 X_1) * \dots * \exp(\alpha_n X_n)$
- ① Then  $\exp(\alpha_0)$  is base frequency, while  $\exp(\alpha_i X_i)$  are loadings saying how much the real frequency will fall/increase compared to the base frequency for a particular value of the risk factor  $X_i$
- ① Very easy to interpret and to check the reasonableness of the model
- ① Typical variables :
  - ① For the car insurance: age of driver, the power of engine, age of car, experience of the driver, number of claims in the past, address of the driver, value of the car...
  - ① For property insurance: age of a building, number of storeys, areal, construction type, type of roof, number of storeys underground, number of people in the household, location, number of claims in the past, is it rented out or not....

# From regression to pure premium

- Similar regression can be made for average claim severity (note that depending on the claim distribution we may have different “link” between regressors  $X_1, \dots, X_n$  and  $E(Z_1|X_1, \dots, X_n)$ )
- Then we use the result that

$$\pi^{pu} = EX = \mu_i T * E(Z_1) = T E(N|X_1, \dots, X_n) E(Z_1|X_1, \dots, X_n)$$

# Compound Poisson with unknown intensity



Book: chapter 6, section 6.2.5  
and 6.3.5



# Compound Poisson model revisited

So far we assumed that  $\text{Claims} = X = \sum_{i=1}^N Z_i$ , where  $N$  random variable with values  $0, 1, 2, \dots$  and  $Z_i$  i.i.d., independent of  $N$ . Moreover usually we assumed that  $N \sim \text{Poiss}(\lambda)$  and we assumed that we know  $\lambda$ .

What happens if  $\lambda$  is unknown? We get then a model (compound Poisson with intensity uncertainty):

1.  $X = \sum_{i=1}^N Z_i$ ,
2.  $Z_i$  i.i.d.,
3.  $N \sim \text{Poiss}(\lambda)$ , where  $\lambda$  is a random variable independent of  $Z_i$ 's and  $N$
4.  $N$  is independent of  $Z_i$ 's

# Moments of such model – via conditional expectation



By rule of double expectation:

$$E(X) = E(E(X|\lambda)) = E(\lambda E(Z_i)) = E(\lambda) E(Z_i)$$

By rule of double variance:

$$\begin{aligned} \text{Var}(X) &= E(\text{Var}(X|\lambda)) + \text{Var}(E(X|\lambda)) = \\ &= E[\lambda(\text{Var}(Z_i) + (EZ_i)^2)] + \text{Var}(\lambda E(Z_i)) = \\ &= (\text{Var}(Z_i) + (EZ_i)^2)E(\lambda) + \text{Var}(\lambda)(EZ_i)^2 \end{aligned}$$



# Lets look at the ratio sd/E

Assume now that  $\lambda = J\mu T$ , where  $\mu$  is a random variable, independent of N and  $Z_i$ 's

If we look at the ratio of standard deviation over expectation, we get

$$\begin{aligned}\frac{sd(X)}{E(X)} &= \frac{\sqrt{JT(Var(Z_i)+(EZ_i)^2) E(\mu) + J^2 T^2 Var(\mu)(EZ_i)^2}}{JTE(\mu) E(Z_i)} = \\ &= \sqrt{\frac{JT(Var(Z_i)+(EZ_i)^2) E(\mu)}{J^2 T^2 (E(\mu)E(Z_i))^2} + \frac{J^2 T^2 Var(\mu)(EZ_i)^2}{J^2 T^2 (E(\mu)E(Z_i))^2}} = \\ &= \sqrt{\frac{(Var(Z_i)+(EZ_i)^2)}{JTE(\mu)(E(Z_i))^2} + \frac{Var(\mu)}{(E(\mu))^2}}\end{aligned}$$



# Conclusion

For Compound Poisson model with uncertainty regarding the intensity of the Poisson variable, the risk per unit of exposure can be decomposed into

- ① a diversifiable risk which can be reduced to zero by increasing the number of policies in the portfolio
- ② and undiversifiable risk, which cannot be diversified away and is connected to the uncertainty of the intensity parameter.