

STK3505

Mandatory assignment 1 of 1

Submission deadline

Thursday 6th NOVEMBER 2025, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have one attempt to pass the assignment. This means that there are no second attempts.

For courses on bachelor level, you can choose between scanning handwritten notes or using a typesetting software for mathematics (e.g. LaTeX). Scanned pages must be clearly legible. For courses on master level the assignment must be written with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. Remember to include any relevant programming code and resulting plots and figures, in the PDF-file.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Problem 1. (30p) A fixed-coupon bond is a financial instrument characterised by maturity T -years, coupon c (which is assumed to be paid out annually) and principal P . For such a bond, a bond holder gets cP amount of money at the end of each year before maturity, and at the maturity they get $cP + P$. The value of the bond at time $t = 0$ is equal to the present value of the underlying cashflows.

1. Assume that the interest rate at time $t = 0$ is flat and equal r . Write down a formula for calculating the value of the bond at time $t = 0$ given the parameters r, T, c, P . (7p)
2. Assume that $r = c$. Prove that the value of the bond is equal to P regardless c, P, T . (5p)
3. Write a function which calculates the value of the bond, given parameters r, c, P, T . Run the function for $r = 5\%, c = 4\%, T = 10$ and $P = 1000$. Run the function again changing different parameters and plot the results (parameter against the value of the bond), first with the interest rate ($r = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$), then coupon ($c = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$) and at last, the maturity of the bond ($T = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20$). What does the results tell you about the relation between different level of parameters and the value of the bond? Why those values are different? (10p)
4. Calculate the duration of the bond for $r = 5\%, c = 4\%, T = 10$ and $P = 1000$ and give an interpretation to it. Assume that the interest rate has increased from $r = 5\%$ to $r = 6\%$. How much would the price of the bond change, as prescribed by the duration? What is the real change in the value of the bond? Why those two value are different? (8p)

Problem 2. (30p) We assume that a random sample $Z_1, \dots, Z_n$ is Rayleigh-distributed. The density, expected value and variance of the Rayleigh distribution is given by:

$$f(X) = \begin{cases} \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) & x > 0 \\ 0 & \text{otherwise} \end{cases},$$

$$\mathbb{E}[X] = \sigma\sqrt{\pi/2}, \quad \text{Var}[X] = \frac{4 - \pi}{2}\sigma^2.$$

1. Derive an inverse sampler for the Rayleigh-distribution (Note: You are expected to compute first the CDF by integration by substitution, then the inverse of CDF). (5p)

2. Derive the moment estimator of σ . (5p)
3. Write a log-likelihood function $L(\sigma)$. Derive the maximum likelihood estimator of σ . (5p)
4. Write a script for a function which calculates the maximum likelihood estimators given a sample $Z_1, \dots, Z_n$. Function should be general enough to handle different n . (5p)
5. Generate $n = 10, 100, 1000, 10000, 1000000$ simulations using the inverse sampler (simulations for different n should be drawn independently) for parameters $\sigma = 3$. Compute a moment estimator σ_{MM} and maximum likelihood estimator σ_{ML} for each n using a function from the previous points and prepare a table presenting the estimated parameters for different n . Compare it to the true value of parameter and comment on the estimation error. (5p)
6. Using a closed-form formula compute mean, standard deviation and 0,025 and 0,975-quantiles for the Rayleigh distribution with estimated parameters σ_{MM} and σ_{ML} for each n estimated in the previous point, as well as for true value of the parameter. Present the values in the table and comment on them. (5p)

Problem 3. (40p) We assume the compound Poisson model for $J = 1000$ policies. In this model, the claim number from all policies is modelled as a random variable $N \sim Poiss(\lambda)$, ($\lambda = J\mu$, $\mu = 0.1$) and the claim size Z_i is distributed according to Weibull distribution with parameters $\alpha = 2, \beta = 3$. Then the sum of all claims can be defined as $X = \sum_{i=1}^N Z_i$.

1. Compute the expectation and the standard deviation of X using the closed-form formula. Calculate the pure premium for one policy and compare it with the expected value of X . What is the relation? (10p)
2. Write a program which simulates the sum of claims X and simulate $n = 10000$ simulations. Compute the mean and standard deviation from the simulation and compare it with point 1. (10p)
3. Assume excess of loss reinsurance, where the reinsurer covers the loss between 2.5 and 5. Write a program which would simulate reinsurance recoverables as well as sum of net claims in this case. (10p)
4. Calculate the pure reinsurance premium in this case as well as mean of the sum of net claims. (10p)