

2.11

$G(X)$ distribution when $X \sim F(X)$ is confined to some subinterval (a, b) . (Truncated X)

a) Show that $G(x) = \frac{F(x) - F(a)}{F(b) - F(a)} \quad a < x < b$

b) Show $G(x) = u$ and show

$$G^{-1}(u) = F^{-1} \{ F(a) + (F(b) - F(a))u \}$$

a) Let $x \in (a, b)$,

$$G(x) = P(X \leq x \mid a < X < b)$$

$$= \frac{P(X \leq x, a < X < b)}{P(a < X < b)}$$

$$= \frac{P(a < X \leq x)}{P(a < X < b)} = \frac{F(x) - F(a)}{F(b) - F(a)}$$

b) $G(x) = u \Leftrightarrow \frac{F(x) - F(a)}{F(b) - F(a)} = u$

$$\Leftrightarrow F(x) = F(a) + u(F(b) - F(a))$$

$$\Leftrightarrow x = F^{-1} \{ F(a) + u(F(b) - F(a)) \}$$

Additional assumption F is strictly increasing.