

2.19

$$R = e^{1 + \gamma_0 \sqrt{z} \varepsilon} - 1 \quad \varepsilon \text{ and } z \text{ indep.}$$

and $\varepsilon \sim N(0,1)$. Suppose $z = 1/G$ $G \sim \Gamma(\alpha, \alpha)$

a) $\log(L+R)$ is t -distributed

$$\log(L+R) = \log(e^{1 + \gamma_0 \sqrt{z} \varepsilon})$$

$$= \underbrace{1}_{\text{shift}} + \underbrace{\gamma_0 \frac{\varepsilon}{\sqrt{G}}}_{\text{scale } t\text{-distribution (2d)}} \Rightarrow \text{scaled and shifted } t\text{-distribution.}$$

2.20

Alternative model for z in 2.19 is

$$z = e^{\tau h + \tau^2/2} \quad \text{where } h \sim N(0,1) \quad \tau \geq 0$$

a) Recall $z \sim \text{LogN}(\mu, \sigma)$ $\Leftrightarrow \log z \sim N(\mu, \sigma)$
and $E[z] = \exp(\mu + \frac{\sigma^2}{2})$ and $\text{sd}(z) = E[z] \sqrt{e^{\sigma^2} - 1}$

$$E[1/z] = E[\exp(-\tau h - \tau^2/2)] = \exp(-\frac{\tau^2}{2} + \frac{\tau^2}{2}) = 1$$

$\sim N(-\frac{\tau^2}{2}, \tau) \Rightarrow 1/z \sim \text{LogN}(-\frac{\tau^2}{2}, \tau)$

$$\text{sd}(1/z) = E[1/z] \cdot \sqrt{e^{\tau^2} - 1} = \sqrt{e^{\tau^2} - 1}$$

b) $1/\tau$ has the same mean and variance
as in exercise 2.19 when $\tau = \sqrt{\log(L + L/\alpha)}$.

Ex. 1.19 $\tau = 1/G \Rightarrow 1/\tau = G$ but $G \sim \Gamma(\alpha, \alpha)$
 $= E[G] = \frac{\alpha}{\alpha} = 1$ and $\text{Var}[G] = \frac{\alpha}{\alpha^2} = \frac{1}{\alpha}$

In this exercise

$$E[1/\tau] = 1 \text{ and}$$

$$\begin{aligned} \text{Var}[1/\tau] &= L^{\tau^2} - L = \exp\left(\left(\sqrt{\log(L + L/\alpha)}\right)^2\right) - L = \\ &= \exp(\log(L + L/\alpha)) - L = L + L/\alpha - L = L/\alpha \end{aligned}$$