

Introduction to Insurance Mathematics STK3505/STK4505

Formalities

- **Classes:** Wednesdays 8:30-10:00 (5 min break) and Fridays 12:15-14:00.
- **Textbook:** Erik Bølviken “Computation and Modelling in Insurance and Finance”, Cambridge University Press, 2014.
- **Exam:** Desember 3rd at 9:00 (4h). **Location:** Georg Sverdrups hus Undervisningsrom 2.
- **Mandatory assignment (most likely):** October 23th, 14:30 (will come back to it)
- **Website:** <https://www.uio.no/studier/emner/matnat/math/STK3505/h25/index.html>
Presentations, solutions of exercises and other info will be published on the website regularly
- The course will cover the material from the textbook, but will be presented in slightly different order and manner.
- Information on the material covered in each week on the website.
- Usually 3 hours of theory and 1 hour of exercises.
- Most of the exercises will be doing computations and they will be done in R. I will try to use R in combination with Jupyter notebooks.
- At the end of the course I will make clear what is the material required for the exam.

Actuarial Science. What it is it?

Actuarial science is the discipline that applies **mathematical and statistical methods** to assess risk in insurance, pension, finance, investment and other industries and professions. More generally, actuaries apply rigorous mathematics to model matters of uncertainty.

Actuarial science includes a number of interrelated subjects, including **mathematics, probability theory, statistics, finance, economics, financial accounting** and **computer science**.

Historically, actuarial science used **deterministic models** in the construction of tables and premiums. The science has gone through revolutionary changes since the 1980s due to the proliferation of high speed computers and the union of **stochastic actuarial models** with **modern financial theory**.

What is differentiating actuarial science from mathematical finance is the focus on risk sharing and diversification in order to mitigate the adverse effect of events of uncertain occurrence, severity and timing.

Structure of the Course

1. Introduction
2. Finance, Part I
 - Time value of money
 - Monte Carlo
 - Distributions and basic models for assets
3. Non-Life insurance
 - Compound Poisson model and Monte Carlo
 - Premium calculation

Reserving
Reinsurance

4. Life insurance

Life-tables, discounting under mortality
Reserves and premium calculation
Markov chains in life-insurance
Monte Carlo simulation
Modern insurance products

5. Finance, Part 2:

Options and Black-Scholes formula
Price of guaranteed returns in life insurance products via options

I will provide also the chapter section in Bølviken's book that cover the syllabus.

Insurance. What is it and how does it work?

This is Chapter 1, section 1.1 and 1.2.

Basic idea: sharing the risk across many individuals in order to be able to cover unlikely but substantial losses



Works because of the Law of Large Numbers. The basic Mathematica model is as follows:

- $X_1, X_2, \dots, X_n$ losses from policy 1, 2, ..., n (assumed to be i.i.d)
- $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n X_i =: \pi^{pu}$ **pure premium**
- Insurance company in this model does not lose (or profit) almost surely if it charges pure premium and has infinite amount of customers.
- In practice
 - No portfolio is infinite
 - Losses can occur at the same time (Natural catastrophes, etc..)
 - Different policies with different risk (but this is less of a problem by proper risk differentiation)
 - Insurance companies want to have some profit.

- Hence, the premium paid has a loading factor γ such that $\pi = \pi^{p^u}(1 + \gamma)$

Types of insurance

Non-life insurance/Property & Casualty (P&C)

Usually 1-year coverage against unlikely risks with adverse financial effect

- Car insurance
- Property
- Travel insurance
- Workers compensation
- Health related products
- Specialistic covers (pet, marine, agriculture, business interruption...)

Life insurance

Usually **longer term contracts** which protect against events where the uncertainty is both **whether** and **when** the trigger event occurs.

- Death insurance/Term insurance
- Pure endowment insurance
- Endowment insurance
- Pension-“insurance against living for too long time”
- Disability insurance
- Nowadays also: saving and investment without any particular “insurance” element.

Role of an actuary

Pricing actuary

- Building models which predict **risk level given risk factors** (e.g. age of a driver)
- Determining premium level for different groups based on those models.
- Risk selection (what to insure, what not)
- Follow up the exposure to detect changes and update models.

Reserving actuary

- Building models to set up reserves.
- Reserves cover **expected future payments** for claims which have not happened, or which have happened but the amount to be paid is uncertain.
- Compare the estimates from pricing against experience

Capital actuary

- Building models to determine how much capital to hold to cover **unexpected losses**, i.e., future payments which exceed reserves.
- Advising about the risk appetite (i.e. how much risk to take given available capital) and how to use capital efficiently.

Finance and the actuarial science

Why finance?

- Reserves can be seen as cashflows (i.e. a sequence of future payments/money transfers happening at different times). Therefore, it is natural to treat them in financial terms and, for example, discount them.
- Investments- appropriate to the nature of the reserves and contracts.
- Interaction between reserves and investments returns, specially for life insurance:
 - Cashflows may depend on the result from investments. Then, the amount of the cashflow can be seen as a financial derivative.

Life-insurance products

Term insurance: Pays an agreed amount (benefit) if insured dies in an agreed period of time. This is an insurance against dying. It is uncertain if the benefit is paid and when.

Pure endowment: Pays a benefit if the insured is alive at the end of an agreed period. This is an insurance against surviving. It is uncertain if the benefit is paid, but no uncertainty about timing.

Endowment: Combination of term and pure endowment. Pays a benefit regardless if insured dies or survives. There is no uncertainty regarding the occurrence of the benefit, but only about timing.

Endowment insurance as savings/investment

- Assume single premium $\pi = 1000$ paid at the beginning of the contract and benefit $B = 1300$, paid out if an insured dies before the next 15 years or after 15 years if an insured survives.
- If the insured survives, then she will yield an annually compounded return $\sqrt[15]{\frac{B}{\pi}} - 1 = 1.76\%$.
- If the insured dies after 5 years, then her heirs will yield an annually compounded return equal to $\sqrt[5]{\frac{B}{\pi}} - 1 = 5.39\%$
- Risk sharing here is between those who survive and get possibly lower returns, and those who die and whose heirs get higher returns.