Introduction to Insurance Mathematics (STK 3505/4505)
Exercises 1

Problem 1 Consider a perpetuity depending on the parameters

m number of payments per year

q number of payment increases per year.

More precisely, the cash flow scheme of the perpetuity is given by

time (in years)				payment
$0,$	$\frac{1}{m},$	$\cdots$	$\frac{1}{q}-\frac{1}{m}$	$\frac{1}{mq}$
$\frac{1}{q},$	$\frac{1}{q}+\frac{1}{m},$	$\cdots$	$\frac{2}{q}-\frac{1}{m}$	$\frac{2}{mq}$
$\frac{2}{q},$	$\frac{2}{q}+\frac{1}{m},$	$\cdots$	$\frac{3}{q}-\frac{1}{m}$	$\frac{3}{mq}$
$\frac{3}{q},$	$\frac{3}{q}+\frac{1}{m},$	$\cdots$	$\frac{4}{q}-\frac{1}{m}$	$\frac{4}{mq}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Derive a formula for the present value $\left(I^{(q)}\ddot{a}\right)^{(m)}_{\infty\rceil}$ of this payment stream.

Problem 2 Let $T = n \in \mathbb{N}$ denote the duration of an annuity.
(i) Show that

$$a_{n\rceil} \;:=\; \frac{1-v^n}{i} \text{ (PV of an immediate annuity with annual payments of 1 starting in 1),}$$

$$a^{(m)}_{n\rceil} \;:=\; \frac{1-v^n}{i^{(m)}} \text{ (PV of an immediate annuity with payments} \frac{1}{m} \text{ } m \text{ times per year).}$$

(ii) Consider the *accumulated value* of an immediate annuity $s_{n\rceil}$, that is

$$s_{n\rceil} := (1+i)^n a_{n\rceil}.$$

Verify that

$$\frac{1}{a_{n\rceil}} = \frac{1}{s_{n\rceil}} + i.$$

Problem 3 (Repayment of a debt) Let S be the present value of a debt that is to be repaid by payments $r_1, ..., r_n$ at the end of the years $k = 1, 2, ..., n$. Further denote by S_k the *principal outstanding*, i.e. the value of the remaining debt at time k immediately after r_k has been paid (dvs. verdien av restgjeld på tid k etter avdrag).

Show that

$$r_k = iS_{k-1} + (S_{k-1} - S_k),\ k = 1, ..., n,$$

that is each payment can be represented as the sum of the interest on the running debt and the *reduction of principal* (dvs. amortisajon av restgjelden).

Problem 4 A *bond* is a contract which gives the *bondholder* (or *lender*) the right to demand from the *bond issuer* (or *borrower*) a series of future payments. The cash flow of the bond is given by a payment of cF at the end of each period and a final payment C at maturity $T = N$.Here F denotes the *face value* and c stands for the *coupon rate*. C is called the *redemption value*, which usually equals F.The present value PV of the bond can be evaluated as

$$PV = cF\frac{1-(1+i)^{-N}}{i} + C\,(1+i)^{-N}\,,$$

where $i \geq 0$ is the *yield to maturity* (effective per period) required by the lender.

Consider a company which is supposed to retire a bond issue with five annual payments of 15000. The first payment is due to December 31, 2015. In order to accumulate the funds, the company begins making annual payments of X on January 1, 2006 into an account paying effective annual interest of 6%. The final payment is due to January 1, 2015. Determine X.

Problem 5 John pays 98.51 for a bond that is due to mature for 100 in one year. It has coupons at 4% convertible semiannually. Calculate the annual yield rate convertible semiannually.