Introduction to Insurance Mathematics (STK 3505/4505)
Exercises 10

Problem 1 Recall Problem 4, Exercises 9: Consider two portfolios of fire insurance policies with total claim amounts described by

$$S_i(t) = \sum_{j=1}^{N_i(t)} X_j^{(i)},\ i = 1, 2,$$

where the claim numbers $N_1(t)$ and $N_2(t)$ are modelled by homogeneous Poisson processes with intensities $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = \frac{2}{3}$, respectively. The claim numbers $N_i(t)$ are assumed to be independent of the $i.i.d$ fire losses $(X_j^{(i)})_{j\geq 1}$ for each $i = 1, 2$. Further suppose that the total claim amounts are independent and that the claim size distributions are specified by

$$\begin{array}{lll} P(X_j^{(1)} = 0) = 0 & P(X_j^{(1)} = 1) = \frac{1}{3} & P(X_j^{(1)} = 3) = \frac{2}{3} \\ P(X_j^{(2)} = 0) = 0 & P(X_j^{(2)} = 2) = \frac{1}{2} & P(X_j^{(2)} = 4) = \frac{1}{2} \end{array}$$

for all $j \geq 1$. The claim size values are expressed in millions of Danish Krone.

Use the Panjer recursion scheme (Theorem 3.3.10 in Mikosch) to compute the exact distribution of the aggregated total claim amounts, that is

$$P(\widetilde{S}(t) \leq n),$$

where

$$\widetilde{S}(t) = S_1(t) + S_2(t)$$

for $n = 0, ..., 6$ in millions of Danish Krone and $t = 1$.

Problem 2 Consider a portfolio of fire insurance policies with total claim amount given by

$$S = \sum_{j=1}^{N} X_j.$$

at time $t = 1$. Suppose that the claim numbers N are binomially distributed with parameters $n = 4$ and $p = \frac{1}{3}$. Further assume that N is independent of the $i.i.d$ fire losses $(X_j)_{j\geq 1}$, whose common distribution is specified by

$$P(X_j = 0) = 0 \quad P(X_j = 1) = \tfrac{3}{5} \quad P(X_j = 2) = \tfrac{3}{10} \quad P(X_j = 4) = \tfrac{1}{10}$$

for all $j \geq 1$. The claim size values are expressed in millions of Danish Krone.

Apply the Panjer recursion scheme (Theorem 3.3.10 in Mikosch) to compute the exact distribution of the total claim amount S, that is

$$P(S \leq n)$$

for $n = 0, ..., 6$ in millions of Danish Krone.