Introduction to Insurance Mathematics (STK 3505/4505)
Exercises 3

Problem 1 A 3-year term life insurance to (x) is defined by the following table:

year t	death benefit	q_{x+t}
0	3	0.20
1	2	0.25
2	1	0.50

Given the discount factor $v = 0.9$, the death benefits are payable at the end of the year of death and the expected present value of the death benefit is Π. Calculate the probability that the present value of the benefit payment that is actually made will exceed Π.

Problem 2 Let $\overline{A}_x$ be the net single premium of a whole life insurance with a payment of 1 unit at the instant of death. As shown in Section 4.4 in the manuscript $\overline{A}_x$ is described by the differential equation

$$\delta \cdot \overline{A}_x = \frac{d}{dx}\overline{A}_x + \mu_x(1 - \overline{A}_x). \tag{1}$$

Find a solution of (1).

Problem 3 Given the following data at the technical interest rate $i = 0.03$:

x years	$\ddot{a}_x$
72	8.06
73	7.73
74	7.43
75	7.15

Calculate the survival probability p_{73}.

Problem 4 Given the following information for a 3−year temporary life annuity-due (Example 5.1.2), contingent on the life of (x) :

t years	payment	p_{x+t}
0	2	0.80
1	3	0.75
2	4	0.50

and $v = 0.9$. Calculate the variance of the present value of the indicated payments.

Problem 5 (Estimation of death probabilities) Assume that a certain group of lives (e.g. policyholders) between the ages of x and $x+1$ has been under observation for a certain period (one or more calendar years). Suppose that n lives contribute to this observation. Let us assume that life no. i is observed between the ages $x + t_i$ and $x + s_i$, $0 \leq t_i < s_i \leq 1$.

For example, a life buying an insurance policy at the age $x + 0.5$ enters the observation and leaves the observation at the age $x + 7.5$ because of policy termination or death.

The sum

$$E_x := \sum_{i=1}^{n} (s_i - t_i)$$

is called *exposure*. In the sequel let D_x the number of deaths during this observation period. Further denote by I the set

$$I := \{i \in \{1, ..., n\} : \text{ life } i \text{ dies during the observation period}\}.$$

Let us also require that

$$\mu_{x+u} = \mu_{x+\frac{1}{2}}, 0 \leq u \leq 1, x = 0, 1, 2, ...$$

See Section 3.5 in the manuscript (or Section 2.5 in Gerber).

Assume that the future life times $T_1(x + t_1), ..., T_1(x + t_n)$ of the above lives are independent. Then the "probability" for the occurence of the above observation is given by the following likelihood function:

$$L = \prod_{i \in I} (\mu_{x+s_i} \cdot_{s_i - t_i} p_{x+t_i}) \cdot \prod_{i \notin I} {}_{s_i - t_i} p_{x+t_i}$$

Note that $\frac{d}{dt} {}_t q_{x+t_i} \big|_{t=s_i-t_i} = \mu_{x+s_i} \cdot_{s_i - t_i} p_{x+t_i}$.

(i) Show that the maximum likelihood estimator for $\mu_{x+\frac{1}{2}}$ is given by

$$\widehat{\mu}_{x+\frac{1}{2}} = \frac{D_x}{E_x}.$$

(ii) Ten lives enter observation at age x. Two lives enter observation at age $x + 0.4$. Two lives leave observation at age $x + 0.8$, one leaves at age $x + 0.2$ and one leaves at age $x + 0.5$. There is one death at age $x + 0.6$. Calculate the maximum likelihood estimate of the force of mortality.