# Introduction to Insurance Mathematics (STK 3505/4505)
# Exercises 4

**Problem 1** A premium for a whole life insurance of 1 (Section 4.1), payable at the end of the year of death, is issued to a life aged $x$. A premium of $G$ is due at the beginning of each year, provided the insured survives.

Given:

(i) $L =$ the total loss of the insurer, when $G = P_x$ (net annual premium).

(ii) $L^* =$ the insurer's loss when $G$ is chosen such that $E[L^*] = -0.20$.

(iii) $Var[L] = 0.30$.

Compute $Var[L^*]$.

**Problem 2** Consider the following policy with a premium refund: A whole life insurance issued to a life aged $x$ provides a payment of $10,000$. Annual premiums are paid at the beginning of the year for the time span of 20 years. Death claims are paid at the end of the year of death. A premium refund feature is in effect during the premium payment period which provides that one half of the last premium paid to the company is refunded as an additional death benefit. Verify that the net annual premium is equal to

$$\frac{10,000A_x}{(1+\frac{d}{2})\ddot{a}_{x:\overline{20|}} - (1 - v^{20} \cdot {}_{20}p_x)/2},$$

where $\ddot{a}_{x:\overline{20|}}$ is the net single premium of a 20-year temporary life annuity-due (Section 5.1) and $d$ the discount rate.

**Problem 3** Given:

(i) $i = 0.10$.

(ii) $a_{30:\overline{9|}} = 5.6$ ($9-$year immediate life annuity of a life aged $x = 30$).

(iii) $v^{10} \cdot {}_{10}p_{30} = 0.35$.

Evaluate $1000P^1_{30:\overline{10|}}$.

**Problem 4** Consider the $3-$year endowment with payments $c_{k+1}$ specified by

| $k$ years | $c_{k+1}$ | $q_{x+k}$ |
|---|---|---|
| 0 | 2 | 0.20 |
| 1 | 3 | 0.25 |
| 2 | 4 | 0.50 |

Annual net premiums of 1 are paid at the beginning of each year while the insured is alive. The endowment amount is equal to the net premium reserve for year 3. The effective annual interest rate is $i = 1/9$. Calculate the end of policy year reserves ${}_{k+1}V, k = 0, 1, 2$.

Hint: Use the recursion formula

$${}_kV + \Pi_k = v\left[c_{k+1} \cdot q_{x+k} + {}_{k+1}V \cdot p_{x+k}\right].$$

See Section 6.

**Problem 5** Given:

| $k$ duration | $\ddot{a}_{\overline{k}\rceil}$ | ${}_{k-1\rvert}q_x$ |
|---|---|---|
| 1 | 1.000 | 0.33 |
| 2 | 1.930 | 0.24 |
| 3 | 2.795 | 0.16 |
| 4 | 3.600 | 0.11 |

Calculate the net premium reserve ${}_2V_{x:\overline{4}\rceil}$ for an endowment at the end of the second policy year.

**Problem 6** Use the data of Problem 4 to determine the variance of the loss $\Lambda_1$ allocated to policy year two. See Section 6.7 in Gerber.