Introduction to Insurance Mathematics (STK 3505/4505)
Exercises 6

Problem 1 Consider the following statistical Danish fire insurance data collected at Copenhagen Reinsurance in the period from 01/01/1980 until 12/31/1990:

Year	Sample size	$\widehat{\lambda}$ intensity
1980	166	0.46
1981	170	0.46
1982	181	0.50
1983	153	0.42
1984	163	0.44
1985	207	0.57
1986	238	0.65
1987	226	0.62
1988	210	0.58
1989	235	0.64
1990	218	0.59
Total	2167	0.54

The intensity $\widehat{\lambda}$ is a measure for the annual frequency of fire losses and is estimated by the reciprocal of the annual sample mean of inter-arrival times of claims. The data in the above table indicate a tendency of increasing intensity over the years. In order to capture this phenomenon it is assumed that the claim number process is modelled by an inhomogeneous Poisson process with mean value function which is piecewise linear, i.e. $\mu(t) = \widehat{\lambda}t + b$ in different years.

Compute the expected present value of the claims in the total claim amount, that is

$$E\left[\widetilde{S}(t)\right]$$

with

$$\widetilde{S}(t) = \sum_{i=1}^{N(t)} e^{-rT_i} X_i$$

for the interest rate $r = 0.03$ and $t = 10$ years and mean fire loss $E[X_1] = 2.2$ in millions of Danish Krone.

Hint: Use the relation between inhomogeneous and homogeneous Poisson processes and employ Proposition 2.1.16 in the book of Mikosch.

Problem 2 Assume that T_i are arrival times described by the Cramér-Lundberg model. In a portfolio the claims are not reported at T_i but at $T_i + V_i$ with a delay $V_i \geq 0$. Thus the number of claims *reported* up to time t is given by

$$\widetilde{N}(t) = \sum_{i=1}^{N(t)} 1_{[0,t]}(T_i + V_i).$$

Suppose that the delays V_i are modelled by i.i.d. positive random variables which are independent of (T_i).

Determine the expected value of $\widetilde{N}(t)$ for $\lambda = 1$ and V_i is uniformly distributed on $[0, 1]$.

Problem 3 Suppose that N is a homogeneous Poisson process with jump intensity $\lambda > 0$. As before we denote by $T_i, i \geq 1$ the arrival times (or jump times) of the Poisson process.

Show that

$$P(T_1 \leq x_1, T_2 \leq x_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(w_1, w_2) g(w_1, w_2) dw_1 dw_2$$

for all $0 \leq x_1 \leq x_2 < \infty$, where

$$\begin{aligned} F(w_1, w_2) &: = \mathbf{1}_{\{0 \leq w_1 \leq x_1, 0 \leq w_1 + w_2 \leq x_2\}}(w_1, w_2), \\ g(w_1, w_2) &: = \lambda e^{-\lambda w_1} \lambda e^{-\lambda w_2}, w_1, w_2 \geq 0. \end{aligned}$$