Introduction to Insurance Mathematics (STK 3505/4505)
Exercises 8

Problem 1 Once more as in Problem 1 of Exercises 6 let us have a look at the following Danish fire insurance data collected at Copenhagen Reinsurance in the period from 01/01/1980 until 12/31/1990:

Year	Sample size	$\widehat{\lambda}$ intensity
1980	166	0.46
1981	170	0.46
1982	181	0.50
1983	153	0.42
1984	163	0.44
1985	207	0.57
1986	238	0.65
1987	226	0.62
1988	210	0.58
1989	235	0.64
1990	218	0.59
Total	2167	0.54

Since the intensity $\widehat{\lambda}$ of fire losses in the above table varies over time it is reasonable to model the claim numbers $N(t)$ by an inhomogeneous Poisson process with mean value function which is piecewise linear, i.e. $\mu(t) = \widehat{\lambda}t + b$ in different years.

Calculate the expected value and the variance of the total claim amount

$$S(t) = \sum_{i=1}^{N(t)} X_i$$

for $t = 9$ years, the expected fire loss $E[X_1] = 2.2$ (in millions of Danish Krone) and $Var[X_1] = 1.63$.

Problem 2 Consider claim numbers $N(t)$ whose dynamics is described by a mixed Poisson process with mixing variable $\theta > 0$ such that $Var[\theta] < \infty$. Show that $N(t)$ is *over-dispersed*, that is

$$Var[N(t)] > E[N(t)],$$

for any $t > 0$ with $\mu(t) > 0$.

Hint: For convenience assume that θ takes values in $\mathbb{N}$.

Problem 3 Consider the following real-life fire insurance data collected at Copenhagen

Reinsurance in the period from 09/01/1990 until 10/01/1990:

Date	Loss in DKM
09/08/1990	14.851485
09/11/1990	1.596535
09/15/1990	3.877888
09/19/1990	2.021452
09/20/1990	1.650165
09/27/1990	1.010726
09/27/1990	1.485149
09/30/1990	1.278878
10/01/1990	1.023102
10/01/1990	4.702970

The fire losses in the above table are expressed in millions of Danish Krone. Assume that the dynamics of the total claim amount follows the renewal model.

Employ the above data to calculate for "large" t

(i) the net premium $p_{Net}(t)$ based on the equivalence principle.

(ii) the expected value premium $p_{EV}(t)$ for the safety loading $\rho = 4\%$. How must be the "safety loading" α of the variance principle based premium $p_{Var}(t) = E[S(t)] + \alpha Var[S(t)]$ chosen such that $p_{Var}(t)$ and $p_{EV}(t)$ become equivalent in the sense of

$$\frac{p_{Var}(t)}{p_{EV}(t)} \longrightarrow 1$$

for $t \longrightarrow \infty$.

Problem 4 Compute the mean and the variance of the total claim amount $S(t)$ under the condition that N is a mixed Poisson process with $N(t) = \widetilde{N}(\theta \cdot t)$, where $\widetilde{N}$ is a standard homogeneous Poisson process, $\theta > 0$ is a mixing variable with $Var[\theta] < \infty$, and (X_i) is an i.i.d claim size sequence with $Var[X_1] < \infty$. Show that

$$\frac{p_{Var}(t)}{p_{EV}(t)} \longrightarrow \infty$$

for $t \longrightarrow \infty$.

Compare the latter limit relation with the case when N is a renewal process.