

STK-4051 / STK-9051

Mandatory assignment 2 of 2

Submission deadline

Thursday 23rd of April 2026, 14:30 at Canvas.

Instructions

This assignment follows the policy “submit to pass”, so it will be approved even if you only submit a blank page. You will receive feedback only if you try to solve the exercises yourself. Assignments only generated by Large Language Models (LLMs) will not be evaluated (it means that you can still use LLMs, but you need to re-elaborate the answers yourself if you want any feedback).

You can choose between scanning handwritten notes or using a typesetting software for mathematics (e.g. LaTeX). Scanned pages must be clearly legible. The assignment must be submitted as a single PDF file. Remember to include any relevant programming code and resulting plots and figures, in the PDF-file.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

Application for postponed delivery

If you experience technical problems, or need a postponement of the deadline, you must contact the course teacher before the deadline expires.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

[Read the department's guidelines for mandatory assignments for more information.](#)

GOOD LUCK!

Problem 1. (inspired by Exercise no. 43 of Hjort, 1990)

Consider a random vector $\{X_1, \dots, X_n\}$, where the X_i , $i = 1, \dots, n$ are generated from independent exponential distribution with parameter θ^{-1} .

- (a) Show that the maximum likelihood estimator $\hat{\theta}$ is approximately distributed as $N(\theta_0, \theta_0^2/n)$ when the model is correctly specified, and $N(\theta_0, \sigma_0^2/n)$ when the model is not correct. Here θ_0 is the true mean, σ_0^2 the true variance.
- (b) Consider the parametric bootstrap estimator $\hat{\theta}^*$. Hjort (1990) states that $\hat{\theta}^* \sim \hat{\theta} \chi_{2n}^2/2n$, while the approximate distribution is $N(\hat{\theta}, \hat{\theta}^2/n)$. Show empirically (via simulation) that this is true. What about the situation with the misspecified model?
- (c) Empirically show that $\hat{\theta}^*$ is approximately distributed as $N(\hat{\theta}, \hat{\sigma}^2/n)$ in the case of non-parametric bootstrap. Does the model specification/misspecification plays any role here? Why?
- (d) For both points (b) and (c), set $n = 20$ and compare the confidence intervals obtained theoretically with those obtained using the simple percentile method, the BC_a approach and the nested bootstrap approach.

Problem 2. Consider the number of “great” inventions and scientific discoveries in each year from 1860 to 1959. The data can be found in R (“> `data(discoveries)`”) and come from Delury (1975, pp 315–318).

- (a) Describe in broad generality (give the main idea, without entering into the details) the two algorithms provided by the textbook to determine a suitable block length for bootstrapping dependent data (“subsampling plus bootstrapping”, “jackknifing plus bootstrapping”). In particular, highlight where the “subsampling” and the “jackknifing” happens.
- (b) Choose one of the two algorithms and use it to determine a suitable block length for the time-series of the `discoveries` dataset.
- (b) Both algorithms depend on the initial choice of some parameters: using the book notation, l_0 and m or d (based on the one you chose). Discuss the possible effect of the choice of these parameters and evaluate it empirically in the `discoveries` dataset.
- (c) Estimate the lag-1 autocorrelation of the number of “great” inventions and scientific discoveries and compute bias and standard error using a moving block bootstrap with an appropriate blocks-of-blocks strategy.

References

- Delury, G.E.** (1975). *The World Almanac and Book of Facts, 1975 Edition*, Newspaper Enterprise Association, New York.
- Hjort, N.L.** (1990). STK 392, Bootstrapping and Resampling Theory, Spring 1990. *In: Lecture Notes, Exercises, Obligs, Exam Sets for various courses given by Hjort.*, UiO.