Lecture 1 on spatial geostatistical processes

Example: Covariance functions (p. 6)

(a) Yes!

$$\sum_i \sum_j a_i a_j c(s_i, s_j) = a\left[\sum_i \sum_j a_i a_j c_1(s_i, s_j)\right] \geq 0.$$

RF	Y	$\sqrt{a}\,Y$
Cov	c_1	$a\,c_1$

(b) Yes!

$$\sum_i \sum_j a_i a_j c(s_i, s_j) = \underbrace{\sum_i \sum_j a_i a_j c_1(s_i, s_j)}_{\geq 0} + \underbrace{\sum_i \sum_j a_i a_j c_2(s_i, s_j)}_{\geq 0} \geq 0$$

RF	Y_1	Y_2	$Y_1 + Y_2$
Cov	c_1	c_2	$c_1 + c_2$

Y_1, Y_2 independent.

(c) Yes! (Need other representations to show)

RF	Y_1	Y_2	$Y_1 \cdot Y_2$
Cov	c_1	c_2	$c_1 \cdot c_2$

Y_1, Y_2 independent

Y_1, Y_2 centered

(d) No!

Let $c_2 = 2c_1$ and $c_1(0,0) = 1$

$\Rightarrow c(0,0) = -c_1(0,0) = -1 < 0$

[$\mathrm{Var}[Y(0)] = -1$ ↯]

Example: 1) stationary, and 2) isotropic? (p. 19)

a) $c(s_1, s_2) = \underbrace{C_Y(s_1 - s_2)}_{\text{stationary}} = \underbrace{C_Y^\circ(\|s_1 - s_2\|)}_{\text{isotropic}}$

b) 1) $s_1 - s_2 = (s_{11} - s_{21}, s_{12} - s_{22})$ so

$c(s_1, s_2) = C_Y(s_1 - s_2) \Rightarrow$ stationary

2) $c\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right) = \exp(-1)$

$c\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right) = \exp(-2)$

but $\|(1,0)\| = \|(0,1)\| = 1$.

$\Rightarrow$ not isotropic

c) $c(0,0) = \exp(0) \times \exp(0) = 1$

$c\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right) = \exp(1+1) \times \exp(0) = \exp(2)$

$\Rightarrow$ Not stationary, not isotropic