UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in:	STK4400 — Risk and reliability analysis
Day of examination:	Tuesday May 25th - Wednesday May 26th 2021.
This problem set consists of 2 pages.	
Appendices:	None.
Permitted aids:	All lecture notes. Collaboration is not allowed.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

a) Can you say what is means to be a convex set? What is a convex combination? How about a convex hull?

b) Can you mention some properties of convex sets?

c) Can you define a convex risk measure and give an economic interpretation of the requirements in this definition?

Problem 2

In this problem we consider a multistate system (C, ϕ) with component set $C = \{1, \ldots, n\}$ and structure function ϕ. The state of the ith component is described by a semi-Markov chain $\{X_i(t)\}$, $i \in C$. We also introduce the component state vector $\boldsymbol{X}(t) = (X_1(t), \ldots, X_n(t))$.

The *n-Birnbaum measure* of importance of component i at time t, denoted $I_{NB}^{(i)}(t)$, is given by:

$$I_{NB}^{(i)}(t) = P[\phi(X_i(t), \boldsymbol{X}(t)) \neq \phi(X_i^+(t), \boldsymbol{X}(t))]. \tag{1}$$

The *p-Birnbaum measure* of importance of component i at time t, denoted $I_{PB}^{(i)}(t)$, is given by:

$$I_{PB}^{(i)}(t) = P[\phi(X_i^-(t), \boldsymbol{X}(t)) \neq \phi(X_i(t), \boldsymbol{X}(t))]. \tag{2}$$

a) Explain the notation used in (1) and (2), and give an interpretation of these measures. Can you describe some possible applications of these measures?

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b) Explain why $I_{NB}^{(i)}(t) = I_{PB}^{(i)}(t)$ in the binary case.

c) Let $\boldsymbol{P}^{(i)} = [P_{uv}^{(i)}]$ and $\boldsymbol{Q}^{(i)} = [Q_{uv}^{(i)}]$ denote respectively the forward and the backwards transition probability matrices of the built-in Markov chain for the process $\{X_i(t)\}$, $i = 1, \ldots, n$. Explain briefly why we have:

$$I_{NB}^{(i)}(t) = \sum_{u,v \in S_i} P[\phi(u, \boldsymbol{X}(t)) \neq \phi(v, \boldsymbol{X}(t))] \cdot P[X_i(t) = u] \cdot P_{uv}^{(i)} \quad (3)$$

$$I_{PB}^{(i)}(t) = \sum_{u,v \in S_i} P[\phi(u, \boldsymbol{X}(t)) \neq \phi(v, \boldsymbol{X}(t))] \cdot P[X_i(t) = u] \cdot Q_{uv}^{(i)} \quad (4)$$

d) Let $I_{NB}^{(i)}$ and $I_{PB}^{(i)}$ denote the asymptotic values of respectively the n-Birnbaum measure and the p-Birnbaum measure, $i = 1, \ldots, n$. Give a sufficient condition for when the two measures are equal. (No proofs are required here.)

e) The *n*-Birnbaum measure* of importance of component i at time t, denoted $I_{NB}^{*(i)}(t)$, is defined by:

$$I_{NB}^{*(i)}(t) = E|\phi(X_i(t), \boldsymbol{X}(t)) - \phi(X_i^+(t), \boldsymbol{X}(t))|.$$

The *p*-Birnbaum measure* of importance of component i at time t, denoted $I_{PB}^{*(i)}(t)$, is defined by:

$$I_{PB}^{*(i)}(t) = E|\phi(X_i^-(t), \boldsymbol{X}(t)) - \phi(X_i(t), \boldsymbol{X}(t))|.$$

Discuss briefly the differences between the importance measures $I_{NB}^{(i)}(t)$ and $I_{PB}^{(i)}(t)$ and the importance measures $I_{NB}^{*(i)}(t)$ and $I_{PB}^{*(i)}(t)$.

END