UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: STK4400 — Risk and reliability analysis

Day of examination: April 30, 2025

Examination hours: Oral exam – 45 minutes

This problem set consists of 3 pages.

Appendices: None

Permitted aids: All lecture notes.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

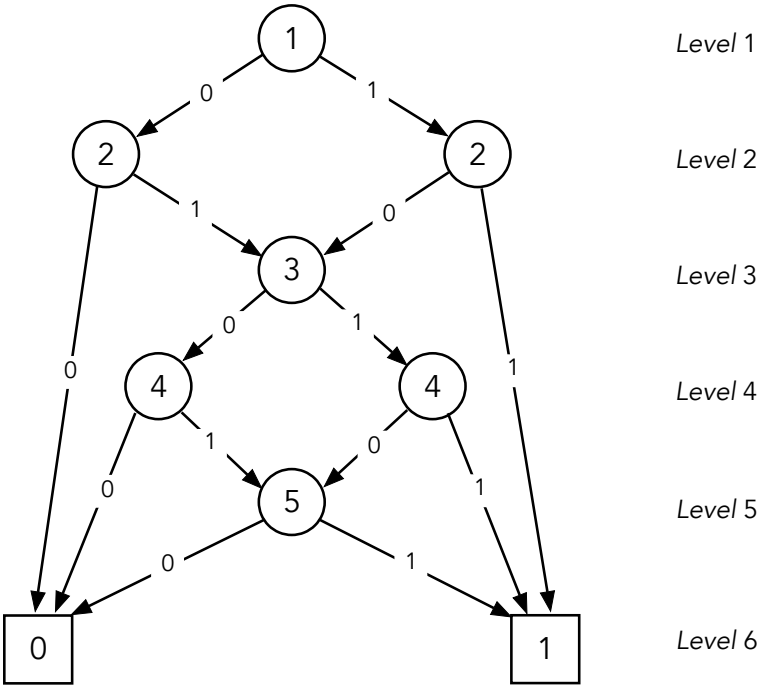


Figure 1: Reduced Binary Decision Diagram

Figure 1 shows a *reduced ordered binary decision diagram* (ROBDD) of a threshold system (C, ϕ) where $C = \{1, 2, 3, 4, 5\}$, and the structure function ϕ is given by:

$$\phi(\boldsymbol{x}) = \mathrm{I}(7x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5 \geq 13) \tag{1}$$

(Continued on page 2.)

In this expression $\mathrm{I}(\cdot)$ denotes the indicator function, $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)$ denotes the vector of binary component state variables where for all $i \in C$:

$$x_i = \begin{cases} 1 & \text{if component } i \text{ is functioning} \\ 0 & \text{otherwise} \end{cases}$$

For each of the circular nodes in the ROBDD we associate a structure function obtained from ϕ by fixing the values of some of the component state variables. More specifically, to the node at level 1, labeled 1, we associate the structure function ϕ itself. To the nodes at level 2, both labeled 2, we associate the structure function $\phi_{-1} = \phi(0_1, \boldsymbol{x})$ to the leftmost node and the structure function $\phi_{+1} = \phi(1_1, \boldsymbol{x})$ to the rightmost node. The structure functions associated with the lower level nodes are derived in a similar fashion.

a) Explain why:

$$\phi_{-1} = \phi(0_1, \boldsymbol{x}) = \mathrm{I}(6x_2 + 5x_3 + 4x_4 + 3x_5 \geq 13)$$
$$\phi_{+1} = \phi(1_1, \boldsymbol{x}) = \mathrm{I}(6x_2 + 5x_3 + 4x_4 + 3x_5 \geq 6)$$

What kind of systems are $(C \setminus \{1\}, \phi_{-1})$ and $(C \setminus \{1\}, \phi_{+1})$?

We let $\phi_{-1-2} = \phi(0_1, 0_2, \boldsymbol{x})$ and $\phi_{+1+2} = \phi(1_1, 1_2, \boldsymbol{x})$.

b) Explain why:

$$\phi_{-1-2} = \phi(0_1, 0_2, \boldsymbol{x}) \equiv 0$$
$$\phi_{+1+2} = \phi(1_1, 1_2, \boldsymbol{x}) \equiv 1$$

How are these relations reflected in the ROBDD?

We let $\phi_{-1+2} = \phi(0_1, 1_2, \boldsymbol{x})$ and $\phi_{+1-2} = \phi(1_1, 0_2, \boldsymbol{x})$.

c) Explain why:

$$(C \setminus \{1, 2\}, \phi_{-1+2}) = (C \setminus \{1, 2\}, \phi_{+1-2})$$

How is this relation reflected in the ROBDD?

HINT: Show that:

$$\phi_{-1+2} = \phi(0_1, 1_2, \boldsymbol{x}) = \mathrm{I}(5x_3 + 4x_4 + 3x_5 \geq 7)$$
$$\phi_{+1-2} = \phi(1_1, 0_2, \boldsymbol{x}) = \mathrm{I}(5x_3 + 4x_4 + 3x_5 \geq 6)$$

Then explain why $5x_3 + 4x_4 + 3x_5 \in \{0, 3, 4, 5, 7, 8, 9, 12\}$, and use this to show that:

$$\mathrm{I}(5x_3 + 4x_4 + 3x_5 \geq 7) = \mathrm{I}(5x_3 + 4x_4 + 3x_5 \geq 6) \quad \text{for all } x_3, x_4, x_5 \in \{0, 1\}.$$

d) Explain how the ROBDD can be used to calculate the reliability of the system (C, ϕ).

(Continued on page 3.)

Problem 2

In this problem we consider a multistate system (C, ϕ) with component set $C = \{1, \ldots, n\}$ and structure function ϕ. We also introduce the component state vector $\boldsymbol{X}(t) = (X_1(t), \ldots, X_n(t))$.

The *n*-Birnbaum measure* of importance of component i at time t, denoted $I_{NB}^{*(i)}(t)$, is defined by:

$$I_{NB}^{*(i)}(t) = E\left|\phi(X_i(t), \boldsymbol{X}(t)) - \phi(X_i^+(t), \boldsymbol{X}(t))\right| \tag{2}$$

The *p*-Birnbaum measure* of importance of component i at time t, denoted $I_{PB}^{*(i)}(t)$, is defined by:

$$I_{PB}^{*(i)}(t) = E\left|\phi(X_i^-(t), \boldsymbol{X}(t)) - \phi(X_i(t), \boldsymbol{X}(t))\right| \tag{3}$$

Explain the notation used in (2) and (3), and give an interpretation of these measures.

END