UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam STK4500/STK9500 v2012: Finans og forsikring

Project assignment, disseminated Friday 1 June 9.00 hrs., deadline for submission Monday 4 June 15.00 hrs.. Submission to be delivered in 2 copies at the secretariat ("ekspedisjonen") at the 7th floor.

Together with the written response to the questions in the assignment, the candidate shall attach a printout of the computer-code used for the assignment's calculations and printout of the slides to be used at the oral presentation.

The candidate shall confirm that his/hers submission is the result of independent work by attaching a dated and signed declaration.

A supplemental 25 minutes oral presentation will take place 13-15 June according to a list which will be published on the course's home page 8 June.

For determining the conclusive grade the written solution to the assignment and the oral presentation will be assessed as a whole, with great emphasis being put on the student's ability to explain and elaborate the written submission.

In this assignment we analyze risk characteristics of defined contribution pension, defined benefit pension and a plan which combines defined contribution and defined benefit (to be defined more precisely and which we will call a hybrid pension plan).

For all types of pension plans, we consider the same financial market that consists of two financial assets. A unit of each of the assets have values respectively B_0 og S_0 at time 0, and the value of the assets at time t, respectively B_t and S_t, are governed by a two-dimensional Geometric Brownian Motion, as follows:

$$B_t = B_0 \cdot exp\left[\left(\mu_B - \frac{\sigma_B^2}{2}\right) \cdot t + \sigma_B \cdot V_t\right]$$

$$S_t = S_0 \cdot exp\left[\left(\mu_S - \frac{\sigma_S^2}{2}\right) \cdot t + \sigma_S \cdot V_t\right]$$

$$(V_t, W_t) \sim N(\begin{pmatrix}0\\0\end{pmatrix}, t\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix})$$

Throughout the assignment we shall use the following parameterization for the description of the financial market:

$$\mu_B = 0{,}05$$

$$\sigma_B = 0{,}05$$

$$\mu_S = 0{,}10$$

$$\sigma_S = 0{,}20$$

$$\rho = 0{,}40$$

The asset with value development S_t is called stock and the asset with value development B_t is called bond.

We consider the employer's cost of funding future pension for a single employee. At time 0 the employee becomes a new member of the company pension plan at age $x = 30$ years. The pensionable age is $x + n = 65$ years. The employee's annual salary at time t, L_t, is governed by the following stochastic dynamics:

$$L_0 = 300\,000$$

$$L_t = (1 + \lambda) \cdot L_{t-1} + \theta \cdot \delta_t \cdot L_{t-1}; t = 1, \dots, n \text{ with } \delta_1, \delta_2, \dots, \delta_n \; i.i.d. \sim N(0,1)$$

$$\lambda = 0{,}03$$

$$\theta = 0{,}015$$

We assume stochastic independence between (V_t, W_t) on the one hand and $\delta_1, \delta_2, \dots, \delta_n$ on the other hand.

For computing premiums and premium reserves for the defined benefit plan we use a valuation basis with discount rate $i = 0{,}03$ (annual) and instantaneous mortality rate ("dødsintensitet") $\nu_{x+t} = \beta \cdot c^{x+t}$ at age $x + t$, where:

$$\beta = 0{,}0000202$$

$$c = 1{,}1015$$

For comparison of the cost of financing future retirement benefits, we calculate the present value at time 0 of the contributions/premiums from inception of membership in the pension plan to the last premium payment. For this present value calculation, we use the discount rate $r = 0{,}06$ (annual).

In the defined contribution plan we assume that the annual contribution is paid in advance in annual terms from inception of pension plan membership until the year before reaching the pensionable age at a rate $100 \cdot p^I$% of annual salary at any point in time. Using P_t^I to denote the contribution at time t, we have:

$$P_t^I = p^I \cdot L_t; t = 0, \dots, n-1$$

We will assume that the contribution rate equals 6%, i.e.:

$$p^I = 0{,}06$$

The defined contribution account is credited by investment return achieved in the financial market and without mortality inheritance ("dødelighetsarv"). Based on the accumulated pension account at pensionable age an annual pension amount, guaranteed life-long and paid in annual terms, is calculated using the same valuation basis as for computing premiums and premium reserves for the defined benefit plan[1].

We assume that the defined contribution account is invested by 20% in stocks and 80% in bonds at any time and that there are no transaction costs of buying and selling these securities.

Spsm. 1. Explain the rebalancing concept that is required to maintain such a fixed relationship 20%/ 80% between stocks and bonds.

Spsm. 2. Using the Monte Carlo simulation approach, find an approximation to the probability distribution of the pension level from the defined contribution plan, measured as the ratio between guaranteed annual pension benefit and annual salary at reaching the pensionable age, and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution?

Spsm. 3. Using the Monte Carlo simulation approach, find an approximation to the probability distribution for the present value of the annual contributions, and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution?

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[1] With a discount rate $i = 0{,}03$ and the financial market and asset allocation as described, there will be "investment surplus" during the course of the pension payment, which could rise to an adjustment to the guaranteed pension amount. Since the focus for this assignment is the impact of the stochastics before the pensionable age, we disregard any such pension adjustment.

In the alternative defined benefit plan we assume that the benefit accrues on a linear/time-proportionate basis measured against current salary at any point in time. With a benefit level of $100 \cdot y^Y$% of current salary, this means that the accrued benefit at time t, O_t^Y, shall be computed as:

$$O_t^Y = \frac{t}{n} \cdot y^Y \cdot L_t; t = 1, \dots, n$$

The funding requirement in the defined benefit plan is as follows: At time t; $t = 1, \dots, n$ the premium reserve shall cover the expected present value of the accrued benefit at time t. Premium payments to comply with the funding requirement is made at time t (e.g. annually in arrears). The premium reserve is credited with return achieved in the financial market and with mortality inheritance ("dødelighets-arv") according to the valuation basis. Negative premiums are allowed.

We will now compare costs between the defined benefit plan and the defined contribution plan as described in the preceding. The level in the defined benefit plan, y^Y, is fixed at the expected benefit level in the defined contribution plan in **Spsm. 2**.

Spsm. 4. Explain possible reasons for negative premiums in the defined benefit plan. What is the practical interpretation of a negative premium?

Spsm. 5. Explain why the same underlying stochastic scenarios for the financial market development and salary development should be used in the comparison.

Spsm. 6. What are the mathematical expressions for the required premium and premium reserve in the defined benefit plan at time t; $t = 1, \dots, n$?

Spsm. 7. Using the Monte Carlo simulation approach, find an approximation to the probability distribution for the present value of the annual premium payments, and illustrate the approximate probability distribution graphically. What is the expectation and standard deviation of the approximate probability distribution? Compare with the expectation and standard deviation in **Spsm. 3** and comment.

Spsm. 8. What is the correlation between the present value of the contribution/premium payments of the defined contribution and the defined benefit plan? Comment.

We now introduce a hybrid pension plan, which combines the properties of defined contribution and defined benefit, abbreviated as "defined contribution with a guaranteed minimum level". The starting point is the same as for a defined contribution plan with a contribution rate of $100 \cdot p^H$% of annual salary at any point in time. At reaching the retirement age if the accumulated defined contribution account is not sufficient to secure a benefit level of $100 \cdot y^H$% of annual salary at that point in time, then the benefit shall be equal to this level. If this guarantee comes into effect, a single premium is paid so that a benefit at a level of $100 \cdot y^H$% of annual salary, guaranteed life-long and paid in annual terms, is fully funded at that point in time.

Spsm. 9. We fix $p^H = 0{,}05$. Using the Monte Carlo simulation approach show that to achieve the same expected benefit level as in the defined contribution plan and the defined benefit plan studied in the preceding, an approximate value for y^H is $0{,}229$. Using the Monte Carlo simulation approach, find an approximation to the probability distribution for the pension level of the hybrid pension plan, and illustrate the approximate probability distribution graphically. What is the probability that the

guarantee of the minimum level $100 \cdot y^H\%$ of annual salary at retirement becomes effective? What is the probability that the benefit level in the hybrid plan is higher than the benefit level in the defined contribution plan?

Spsm. 10. We now assume 30% instead of 20% invested in the stock, while we maintain the assumptions of $p^I = 0{,}06$ and $p^H = 0{,}05$. Find the value of y^H so that the expected benefit level for the hybrid plan is the same as for the defined contribution plan. What is the probability that the guarantee of the minimum level $100 \cdot y^H\%$ of annual salary at retirement becomes effective? What is the probability that the benefit level in the hybrid plan is higher than the benefit level in the defined contribution plan? Compare with the results in **Spsm. 9** and comment.

Spsm. 11. The hybrid plan as it has been defined provides for a minimum guaranteed benefit level only for those members who continue in employment until the retirement age. Discuss possible extensions or modifications of the hybrid pension plan so that the accrued benefit to be secured at termination of employment before the retirement age contains an element of a guaranteed minimum benefit level.