Diversification: Financial risk vs. demographic risk

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Traditional life insurance/pension undertaking

Contractual payments to insured individuals contingent upon:

- remaining life time (annuity)
- time of death (life insurance)
- occurence and potential duration of disability (long term disability pension)
- dependent's remaining life time (survivor's pension)
- etc.

Risk nature

Risk exposure: Random variations associated with biometric events - "demographic risk"/"biometric risk"

Do away with risk *in the aggregate* by sufficiently large portfolio:

- diversification
- law of large numbers

Assumptions:

- homogenous risks
- independent risks

Funding: Basic principle

Policyholders' obligations in return for insurer's obligation:

· Premium payments

· *In advance*

Pre-funding ⇒ Accumulation of funds

Funding: Technical base

Balance between :

· contractual outgoes

· contractual ingoes and investment income

Balance in *expected* terms and *over time*.

$$E\left(\sum \text{Benefits}\right) = E\left(\sum \text{Premiums} + \text{Return}\right)$$

Principle of equivalence

Carrying out principle of equivalence

Mathematical expectation w.r.t. demographic risk well understood and substantiated
control perspective.

Mathematical expectation w.r.t financial risk:

- what is it?
- how does it work?

Financial risk *not diversifiable*

First attempt to manage financial risk

Pretend that financial risk can be disregarded.

Artificial deterministic discount rate: Sufficiently low to be realised "almost certainl

Not very satisfactory:

- Theoretically
- In practice

Deterministic discount rate in risky financial ma

Setting:

· actual return on insurer's investment *is* stochastic, with some probabilistic properti

· insurer has an accrued liability - represented as a future (stochastic) payment strea

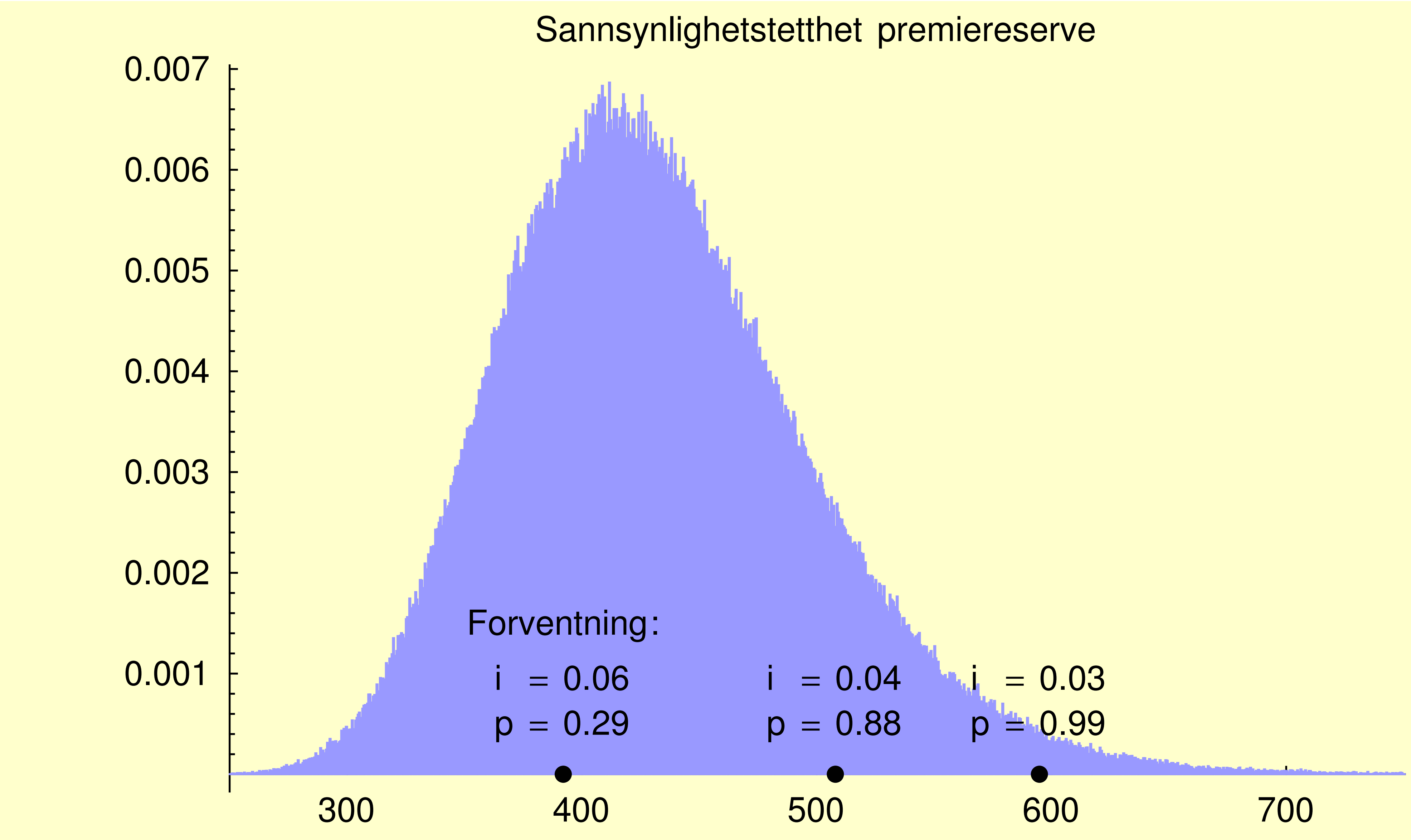
· premium reserve for accrued liability stipulated by discount rate "to the safe side"

Key question: Relation between:

· capital *actually required* to finance insurer's accrued liability - expressed as a prob distribution

· premium reserve - expressed as fixed amount; expected present value as if investn was deterministic

Grafikk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p = 0.29
i = 0.04
p = 0.88
i = 0.03
p = 0.99

Case for considering non-diversifiability of fina risk

Actuarial present value of deferred annuity :

$$P = \sum_{t=k}^{\infty} I[T > t] \cdot v_t$$

where

· T = remaining lifetime for insured individual

· v_t = factor for discounting from time t back to time 0.

Non-diversifiability of financial risk: Basis

Two lifes T^1 and T^2 *i.i.d*.

$$P^i = \sum_{t=k}^{\infty} I\left[T^i > t\right] \cdot v_t; \; i = 1, 2$$

P^1 and P^2 :

· independent if v_t' *s* deterministic

· dependent if v_t' *s* stochastic!

Non-diversifiability of financial risk: Basis

n lifes $T^1, T^2, \ldots, T^n$ $i.i.d.$

$$P^i = \sum_{t=k}^{\infty} I\left[T^i > t\right] \cdot v_t; \; i = 1, 2, \ldots, n$$

Assume v_t's stochastic, whereby all P^i's dependent with :

$$\mathrm{Var}\left(P^i\right) = \sigma^2; \; i = 1, 2, \ldots, n$$

$$\mathrm{Cov}\left(P^i, P^j\right) = \rho \cdot \sigma^2; \; i, j = 1, 2, \ldots, n$$

Then :

$$\mathrm{Var}\left(\frac{1}{n} \sum_{i=1}^{n} P^i\right) =$$

$$\frac{1}{n^2} \sum_{i=1}^{n} \mathrm{Var}\left(P^i\right) + \frac{1}{n} \sum_{i \neq j} \mathrm{Cov}\left(P^i, P^j\right) = \frac{1}{n^2} \cdot n \cdot \sigma^2 + \frac{1}{n} \cdot n \cdot (n-1) \cdot \rho \cdot \sigma^2 = \sigma^2 \cdot \left[\frac{1}{n} + \right.$$

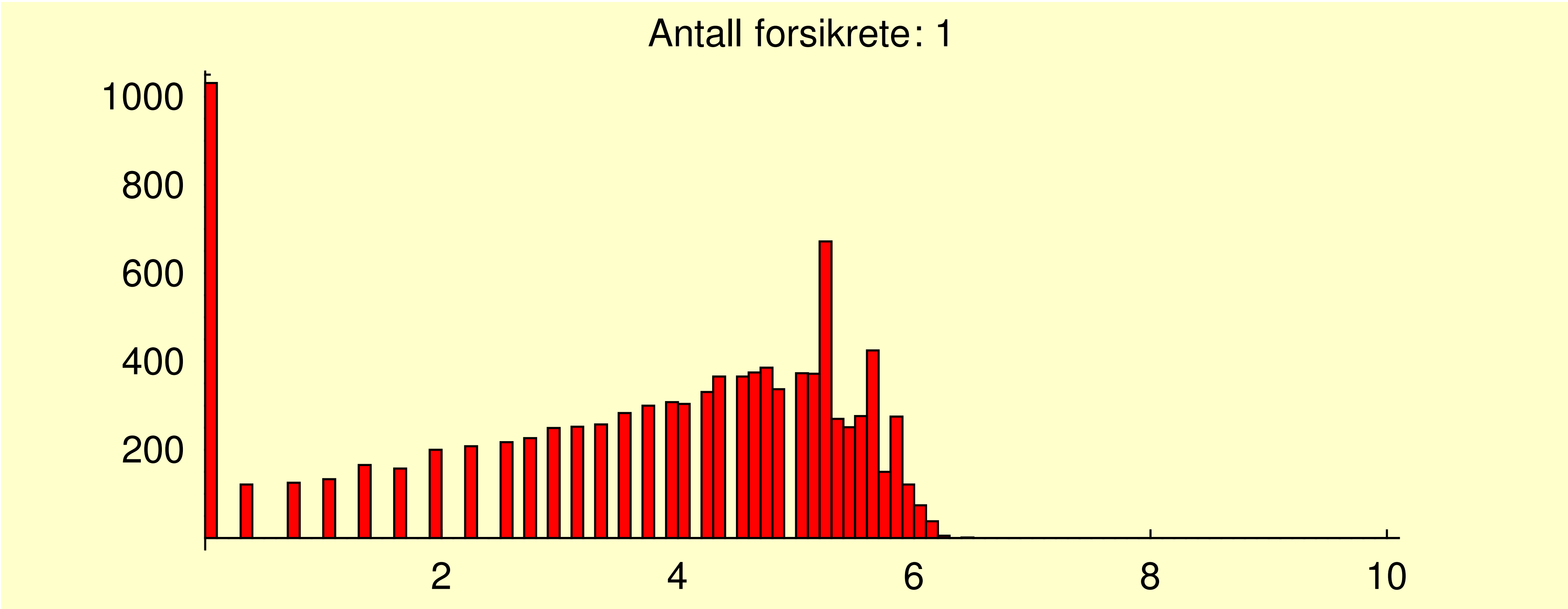
which does not converge to zero as portfolio size increases!

Portofolio uncertainty in the absence of financi

Pdf. for:

$$\frac{1}{n}\sum_{i=1}^{n} P^i$$

under deterministic investment return.

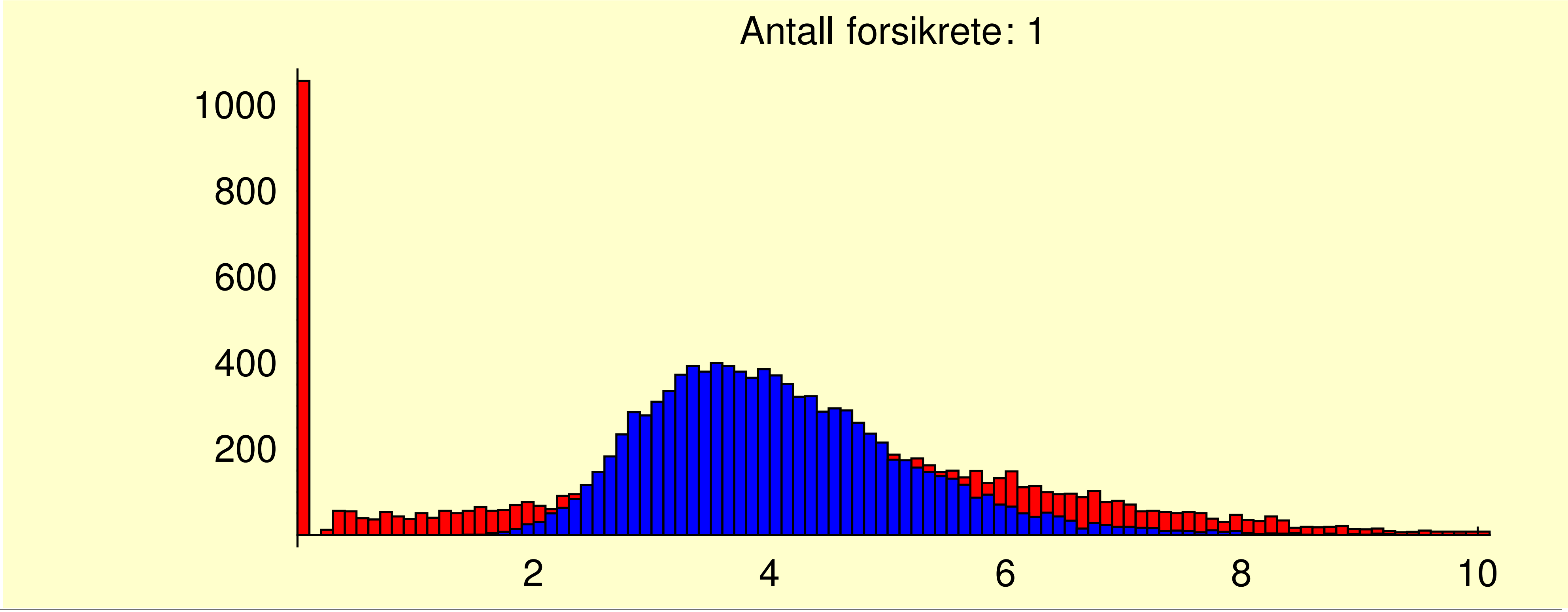


Portofolio uncertainty in the presence of financ risk

Pdf. for:

$$\frac{1}{n}\sum_{i=1}^{n} P^i$$

under stochastic investment return.



Volatility on investment return → variability of annuity's present value.

Simple financial market model :

R_t; $t = 1, 2, ...$ is investment return for period $[t - 1, t)$

Assume $R_1, R_2, ...$ $i.i.d.$, $\mu^R = E(R_t)$ $\sigma^R = \sqrt{\mathrm{Var}(R_t)}$

Discount rates :

$$v_t = \prod_{s=1}^{t} \frac{1}{1 + R_s}; \; t = 1, 2, \; ...$$

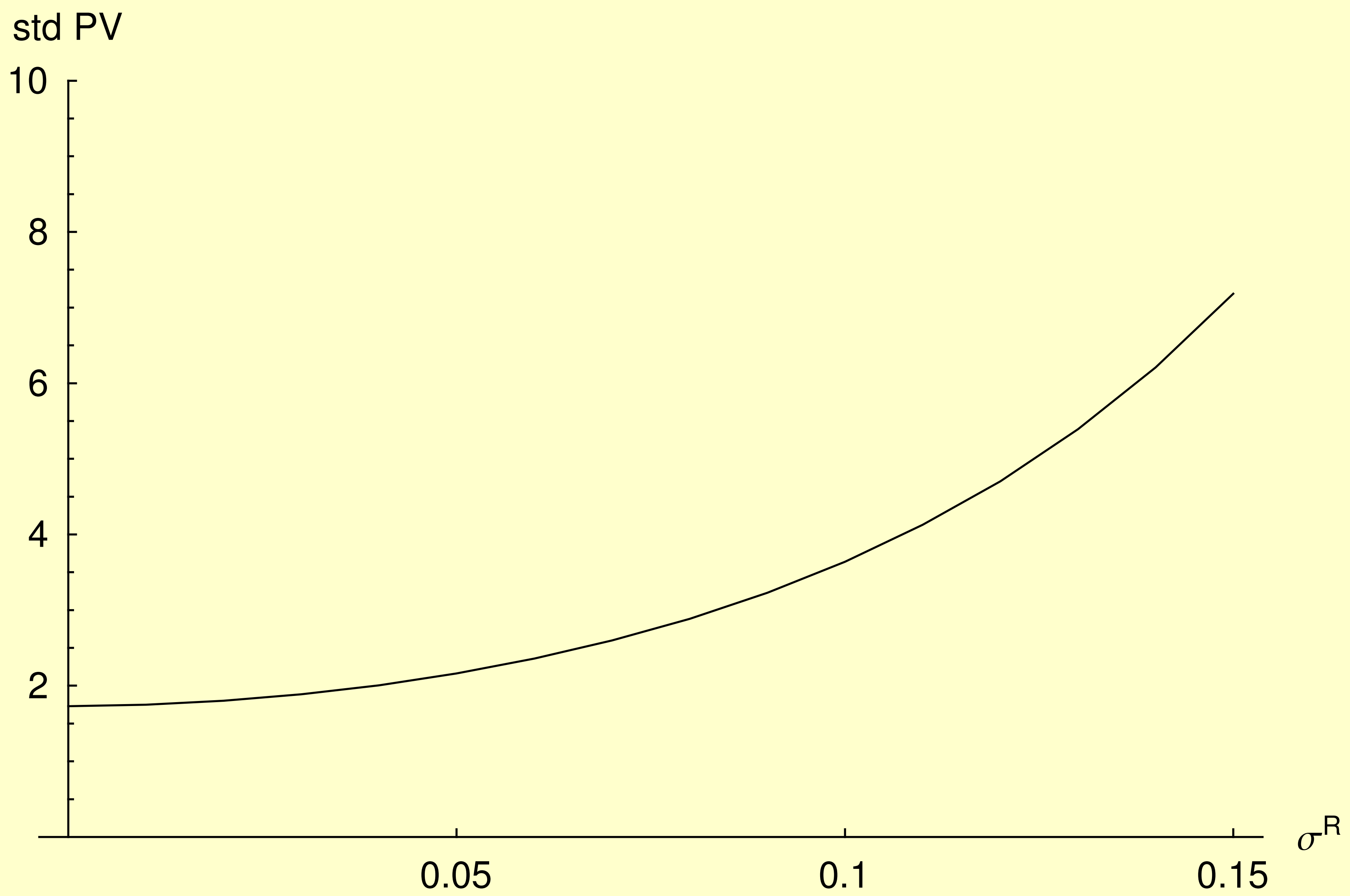
Obtain (approximate) probability distribution for

$$P^i = \sum_{t=k}^{\infty} I\left[T^i > t\right] \cdot v_t$$

by stochastic Monte Carlo simulations of T^i and R_s realisations. (Has in fact already been done for the preceding graphical illustrations).

In particular : How does increased volatility in portfolio / financial markets affect the variability of annuity's present value, as measured by $\mathrm{Std}\left(P^i; \sigma^R\right)$?.

Standard deviation for annuity's PV depending standard deviation for investment return.



Correlation between portfolios.

Two portfolios : $\{T^i\}_{i=1}^n$ and $\{T'^j\}_{j=1}^m$, all $i.i.d$

$$\text{Cov}\left(\sum_{i=1}^n P^i, \sum_{j=1}^m P'^j\right) = \sum_{i,j} \text{Cov}\left(P^i, P'^j\right) = m \cdot n \cdot \rho \cdot \sigma^2$$

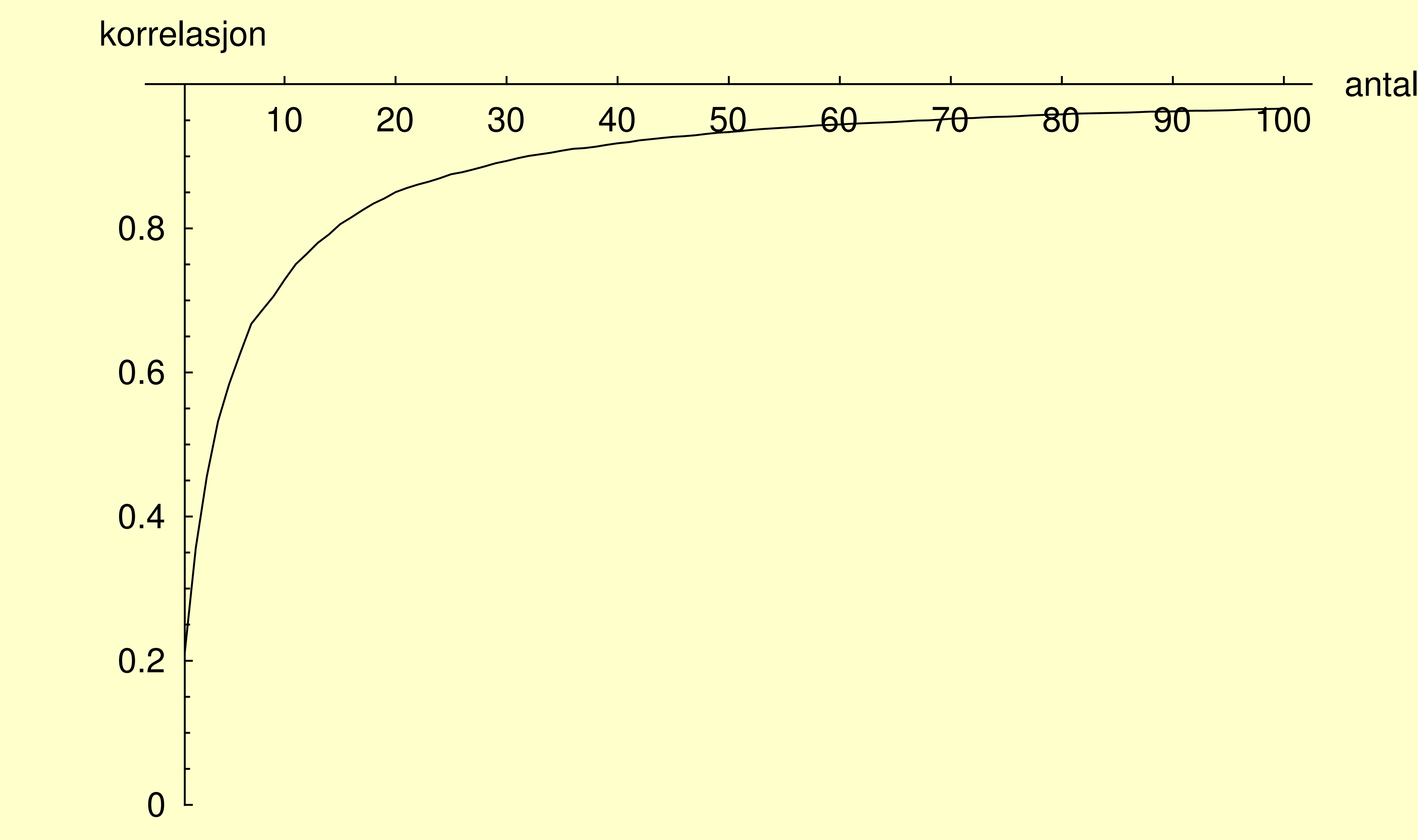
$$\text{Var}\left(\sum_{i=1}^n P^i\right) = n \cdot \sigma^2 + n \cdot (n-1) \cdot \rho \cdot \sigma^2$$

$$\text{Var}\left(\sum_{j=1}^m P'^j\right) = m \cdot \sigma^2 + m \cdot (m-1) \cdot \rho \cdot \sigma^2$$

$n = m$:

$$\rho\left(\sum_{i=1}^n P^i, \sum_{j=1}^n P'^j\right) = \frac{\rho}{\frac{1}{n} + \frac{(n-1)}{n} \cdot \rho} \xrightarrow{n \to \infty} 1$$

Correlation with increasing portfolio size



Modelling and managing financial risk: Founda

Financial market model within probabilistic framework:

- (reasonably) realistic
- operational
- possible to validate and estimate against market data/behaviour

Risk issues:

- recall: non-diversifiable
- how to manage?
    - additional capital
    - investment strategies
    - other?
    - possible to completely eliminate the impact of risk?

Financial risk isolated vs. financial risk and liability risk considered as a whole

Course content

Stochastic modelling of financial assets' market value

Pension fund risk in the presence of financial risk

Derivatives:
- Concept
- Pricing
- Hedging

Application: Interest rate guarantees for pension contracts

Porfolio theory

Matching of assets to liabilities

Riks measures and management

Evaluation - Monte Carlo simulation.

Course content - limitations and extended perspective

Basic concepts!

Extended perspective of great importance for life insurance/pensions undertakings:
- Very long time perspective
- Interest rate sensivity
- Possibilities to hedge/securitise long term interest rate guarantees

Regulatory environement:
- Explicit pricing of interest rate guarantee compulsory in Norway from 2008
- Solvency II: Pension and insurance liabilities valued at "market value"
    - expected cash-flows discounted at market interest rates
    - investment strategy to hedge liability risk?