Oblig STK4500 Spring 2017

Introduction

The oblig carries out Solvency II calculations for an insurance company with a pension portfolio and a term insurance portfolio. Mortality catastrophies have been reinsured away and are not taken into account and there is no disability/morbidity portfolio. You are to report the Solvency II values for the Best Estimate and the Technical Provisions (with zero for 'Other Liabilities') and secondly the solvency capital requirement. All of this is carried out by the Standard Model, but at the end an internal one is introduced, and you are to compare projections under the two evaluations.

You must submit a report either electronically to **erikb@math.uio.no** no later than

April 28, 2017, 16.00 hours

or on paper within the same time limit to **Erik Bølviken, Department of Mathematics, University of Oslo, po box 1053, Blindern, 0316 Oslo**[1].

Cooperation with others are not forbidden, but the report itself **must** be written by you alone, more on what the report should contain at the end.

You must read the note on Solvency II that can be downloaded from the course website prior to the work on this oblig. That material will be lectured on **March 10** and **March 17**, and the oblig itself will be explained on **March 24**. That includes R-software that can be downloaded from the webpage. Questions can be directed to **erikb@math.uio.no**.

Portfolios and introductory calculations

Portfolio 1 is a pension portfolio of equal policies. All the contracts were drawn up at the age of $l_0 = 30$ years and last to the end of life which you define as $l_e = 110$. The retirement age is at $l_r = 67$ years, and the annual pension receved after that date is $s_1 = 0.3$; the money unit is million NOK. Prior to retirement an equivalence premium is contributed (also in advance) at the start of each year. You have to compute π_1 yourself (and report it) using the technical rate of interest $r = 2\%$ and the Gomperz-Makeham mortalities

$$q_l = 1 - e^{-\theta_0 - (\theta_1/\theta_2)(e^{\theta_2}-1)e^{\theta_2 l}}$$

with parameters

$$\theta_0 = 0.00078, \qquad \theta_1 = 0.0000376 \qquad \theta_2 = 0.092759$$

which are representative for US males.

Portfolio 2 is insurance a against death with a one-time payment $s_2 = 2$ (again in million NOK) transferred a beneficiary upon death of the policy holder. Again all the contracts are equal. They were agreed at $l_0 = 30$ years and lasts $K = 25$ years with premia π_2 paid at the start of each year and the benefit (if there is any) transferred immdiately after death has occurred (which means at

[1] There is a post box at the seventh floor of the math building (Niels Henrik Abels hus).

the end of the current year). Again you have to determine π_2 yourself as the equivalence premium under the same conditions as for Portolio 1.

Distribution of individuals What is not equal among policies is how long into the contacts periods they are. There are in Portfolio 1 N_k individuals (all alive) that set up their polices k years earlier where

$$N_{1k} = C_1 \times e^{-\gamma_1 \times |k-\mu_1|}, \qquad k = 0, \ldots, 65.$$

Here $\gamma_1 = 0.15$, $\mu_1 = 5$, and C_1 is determined so that there are approximately 500000 individuals. The distribution of the other portfolio has similar mathematical form. Now

$$N_{2k} = C_2 \times e^{-\gamma_2 \times |k-\mu_2|}, \qquad k = 0, \ldots, 25.$$

where $\gamma_2 = 0.10$, $\mu_2 = 8$, and C_2 determined so that there are, as in the pension portfolio, approximately 500000 individuals.

Other assumptions

It is assumed that the risk-free rate of interest taken from the market is

$$r_k^{\mathrm{rf}} = 0.015 + 0.20(1 - e^{-\theta \times k}) \qquad \text{where} \qquad \theta = 0.15$$

for $k = 0, 1, \ldots$, as long as you need it. The rate of inflation is $I_k = 2\%$ per year.

Standard model calculations

You have to program standard actuarial techniques to determine the Best Estimates BE_1 and BE_2 for the two portfolios. Next come the solvency capital requirements SCR_1 and SCR_2 for the two portfolios which are based on stresses $S_1 = -0.20$ and $S_2 = 0.15$; i.e the Gomperz-Makeham mortalities above are changed to

$$q_l^{s_1} = (1 + S_1)q_l \qquad \text{and} \qquad q_l^{s_2} = (1 + S_2)q_l.$$

When the earlier calculations are repeated under the stressed conditions you obtain revised Best Estimates BE_1^s and BE_2^s from which SCR_1 and SCR_2 are computed. They are merged to the SCR for life insurance risk by the Standard formula with correlation $\rho = -0.25$.

The Internal model

An alternativ way of using the stress models is to apply them both to both portfolios to take into account the natural hedging property when pensions and insurance against death are responsibilities of the same company. A natural formulation of the solvency capital requirement is now

$$\mathrm{SCR}^{\mathrm{intern}} = \max(\mathrm{BE}_1^{s_1} + \mathrm{BE}_2^{s_1}, \mathrm{BE}_1^{s_2} + \mathrm{BE}_2^{s_2}) - (\mathrm{BE}_1 + \mathrm{BE}_2)$$

Argue that this is a more accurate 99.5% percentile than the Solvency II version if S_1 and S_2 define 99.5% percentiles of the mortailities down and up (as assumed in Solvency II).

The report

The report you submit must contain the conclusions of your study and documentation of how they are found. Software put out on the webpage of the course is at your disposal, but the internal model at the end you must program yourself.