Solvency II in life insurance

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1 Introduction.

The European Union has throughout the last decades been setting up the Solvency II system of regulation of the European insurance industry aiming at *generality* (cover everything) and *simplicity* (complex processes approximated by simple mathematical relationships). This is material practitioners need to know, and it also offers university students opportunities to learn how basic insurance concepts are put to work in real life. The purpose of this note (meant to supplement the curriculum of the course STK4500 at The University of Oslo) is to present a simplified extract of the life insurance part of the Solvency II regime. An overview of the general approach is presented in the next section before proceeding to life insurance in Section 3. Readers must be familiar with basic life insurance mathematics, but not all the concepts connected to the financial part which are provided on the way. The concluding Section 4 discusses certain weaknesses in the Solvency II approach and introduces the concept of **Internal** models which when approved by the Insurance Supervisors may replace the **Standard** one treated elsewhere in this note.

A huge pile of documentation of Solvency II exists, and the interested reader can even follow how it has evolved since the turn of the century. The legislation can be looked up the Official Journal of the Europen Union of May 22, 2014, but a more convenient reference for the present purpose is the Commission Delegated Regulation (2015), also known as the 'Delegated Acts' which present specifications and clauses in mathematical form[1.1]. Other sources are EIOPA (2014a,b,c) and EIOPA (2015) which are four manuals with EIOPA(2014a) being the original and principal one with strong overlap with the Delegated Acts. Nothing of this is easy reading. Clauses and rules extend over hundreds of pages, and a reader is stranded in a thick jungle of particular features, important at a later stage, but not in the beginning while trying to cope with the basic ideas and idiosyncracies of the system. The monograph by Sandström (2011), handbook for specialists, has much the same flavour. Another type of references is provided by Deksler et al (2013), Cadoni (2014) and Doff (2014) who probe deaply into practical issues and aspects of Solvency II, but with little or no mathematics. Morin and Thourat (2014) (in French) is a brief summary written in lexiographic style.

2 Solvency II overview.

2.1 The balance sheet.

The top level of the Solvency II balance sheet is shown in Figure 1. On the left there are the assets A which are invested in bonds, equity, property and other financial instruments. Some of them are also held in cash. There are market prices for everything which yield values which can be added for the so-called **fair** (or market) value of the entire financial portfolio. All income a company receives goes into the assets, in particular fees (or premia) from policy holders. Those typically come early in the life of a contract (so that it is the insurer who will own the customers money) and creates **liabilities** for the company. Their valuation is a much more complex task since many of these obligations belong to the future and are by no means completely known. The picture is made still more complicated by the time element with cash flows extending decades into the future.

[1.1] The Delegated Acts consist of articles containing the regulatory rules and will be referred to by their numbers so that, say Article 104 means that article in the Delegated Acts.

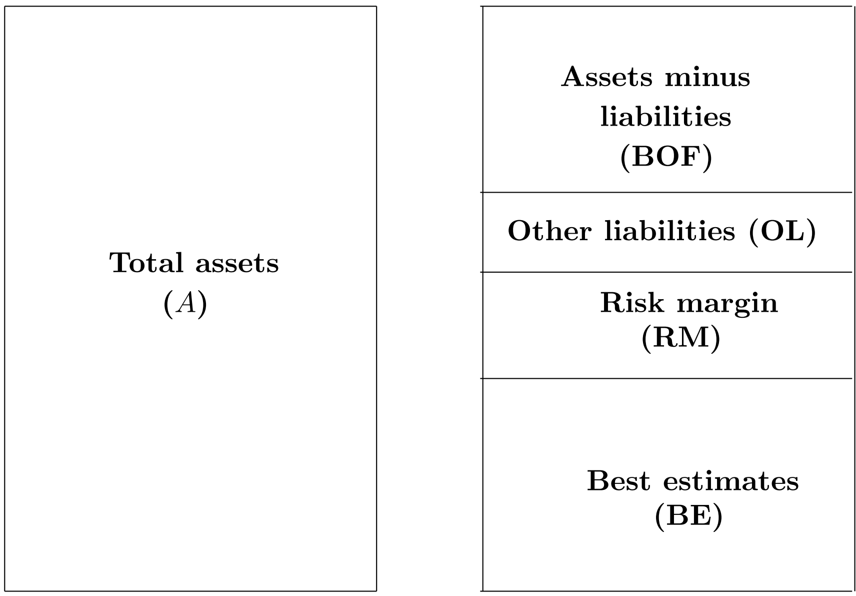

Figure 1 *The Solvency II balance sheet of an insurance company with assets (left) and liabilities (right).*

The complexity of liability valuation is reflected on the right in Figure 1 with the three main Solvency II components. The largest of them in practice is the so-called **Best Estimates** (**BE**) at the bottom right which is the expected values of all future cash flows out of and into accounts under existing contracts. 'Expected' should be interpreted in a mathematical sense, and is in principle an expectation under some underlying stochastic model; more on that later. Future premia are included as are reinsurance compensations, and both are subtracted the amounts needed to liquidate claims and cover expenses so that that BE is an expected, **net** obligation. It makes use of **market risk-free interest rates** for discounting which means that future fluctuations in A on the left in Figure 1 are not independent from those in BE; more in Section 2.4.

Then there is the so-called **Risk margin** (denoted **RM**). It is in Solvency II defined as the extra another insurance company would have to be paid to take over responsibility for all obligations under BE. Note that BE itself would be a break-even price (since it is an expectation), and the new company would charge more. Hence RM>0, but how much larger? There isn't really a market answer to this, yet Solvency II has invented one which is detailed in Section 2.5. The vision is that BE+RM is the market value of all net liabilities.

An insurance company has liabilities beyond those in BE, for example tax obligations that are pending, debt securities and other loan arrangements, employee and share holder benefits or other amounts that have to be paid. All of these are in Figure 1 lumped into **other liabilities** (or **OL**) as in Cadoni (2014), and the total liabilities then become

$$\mathrm{TP} = \mathrm{BE} + \mathrm{RM} + \mathrm{OL} \tag{2.1}$$

which is known as the **Technical Provisions** (hence the abbreviation **TP**). It should be smaller than the value A of the assets; otherwise the company is bankrupt. The difference

$$\mathrm{BOF} = A - \mathrm{TP} \tag{2.2}$$

is known as the **Basic Own Funds** (or BOF for short), and negative BOF must be avoided at all cost which is a principal aim of the Solvency II regulation. OL will in this note be ignored so that $\mathrm{TP} = \mathrm{BE} + \mathrm{RM}$.

2.2 The vision of Solvency II

One part of Solvency II is to report the Technical Provisions and the three components in (2.1), but that does not bring us beyond what is expected and tells nothing about the risk of future insolvency which is what the rest of Solvency II is about. The time horizon is now **one year**. Assets and Technical Provisions are going to change during this period, say through

$$\underset{\textit{today}}{(A_0, \mathrm{TP}_0)} \quad \longrightarrow \quad \underset{\textit{in one year}}{(A_1, \mathrm{TP}_1)} \tag{2.3}$$

with their values TP_0 and A_0 having been turned into TP_1 and A_1 though countless events and operations, for example

- pensions and other obligations paid,
- premia received under old and new contracts,
- new incidents such as death or disability having created new obligations,
- gain (or loss) on investments,
- overhead cost to run the company,
- dividend (to shareholders) and taxes,

and there will be other things too. Suppose it was possible to build a model that describes them all. It would have to be stochastic, and probabilities can then be calculated, say $\Pr(\cdot|A_0)$ emphasizing the relevance of the value A_0 of the assets today. Of course, these probabilities do depend on countless other things (for example, how assets have been invested and what kind of insurance branches we are involved in), but that can remain hidden in the mathematical notation.

Control of insurance companies in Solvency II is carried out through the **Solvency Capital Requirement** (abbreviated **SCR**) which can be defined as the solution of the equation

$$\Pr(A_1 < \mathrm{TP}_1 | A_0 = \mathrm{TP}_0 + \mathrm{SCR}) = \epsilon \qquad \text{where} \qquad \epsilon = 0.005, \tag{2.4}$$

and $\mathrm{TP}_0 + \mathrm{SCR}$ is the minimum value of assets needed to keep the risk of bankruptcy one year later below 0.5%. Implicit in the specification is the assumption of **going concern** which means that the assets are used for what they were built up for and not sold out for quick profit without regard for existing obligations. There is also the additional **Minimum Capital Requirement** defined through the analogy

$$\Pr(A_1 < \mathrm{TP}_1 | A_0 = \mathrm{TP}_0 + \mathrm{MCR}) = \epsilon \qquad \text{where} \qquad \epsilon = 0.15 \tag{2.5}$$

which leads to the capital threshold $\mathrm{TP}_0 + \mathrm{MCR}$ which is a much smaller value than $\mathrm{TP}_0 + \mathrm{SCR}$. It has dramatic consequences for an insurance company if assets fail to comply with the minimum capital requirement; its authorisation is immediately suspended.

Only the solvency capital requirement SCR will be discussed in this note, and it will emerge in Section 3 that Solvency II proceeds through a recursion over an oriented graph. This does not solve the equation (2.4) directly, and although 99.5% solvency is quoted everywhere in Solvency II writings, a mathematical justification is lacking.

2.4 The Best Estimate

One of the key quantities in Solvency II is the Best Estimate BE which is, as already mentioned, the summary of all expected, future cash flows for all existing contracts. Note that there is no notion of prudence or conservatism here; the evaluation seeks the expected as realistically as possible. New business anticipated to be drawn up, say during the next 12 months, is *not* included.

Suppose the activities of the company are followed annually for $k = 1, \ldots, K$ where K is the last year there are any movements in the accounts on the basis of the present business. The mathematical expression for the Best Estimate is then

$$\mathrm{BE} = \sum_{k=1}^{K} \frac{L_k}{(1+r_k)^k} \tag{2.6}$$

with L_k the expected, net cash flow *out of* the company accounts in year k. Money is in nominal (not real) terms, and $L_1, \ldots, L_K$ depend om an assumed rate of inflation I. Included in (2.6) is pensions (ordinary and disability), insurance claims and reinsurer premia paid (counted positive) and premia from clients and reinsurer compensations (counted negative). There are also expenses to run the company (positive) and even expected loss on credit to mortgage holders or reinsurers defaulting on their obligations. (positive again). The discounts are defined in terms of the forward risk-free interest rate curve $r_1, \ldots, r_K$. Is the latter unfamiliar ground? It is published daily in the financial press, and r_k is simply the interest you get by buying a very secure, zero-coupon bond that matures in k years[2.1]. It is Solvency II practice to compute all present values (which BE is) by discounting according to these quantities. If K (which may be up to seven or eight decades) is outside the range of the forward interest rate curve, you must extrapolate it in some way beyond where it is quoted.

2.4 The risk margin

The risk margin RM, the second component in (2.1), is defined by BE+RM being the price a hypotetical second company would have to be paid to take over responsibility for portfolios and obligations. There would be no profit in BE alone so the price must be higher[2.2], and RM is positive. Its valuation is based on the following idea. SCR provides buffer against uncertainty, and keeping capital above what is expected has cost, say (in Solvency II notation) **CoC** (known as the **Cost of Capital**) per money unit[2.3] so that the extra the second company might charge becomes SCR $\times$ CoC.

[2.1] A zero-coupon bond is a security with a single payment that takes place at maturity.

[2.2] The first company has presumably received part of the premia earlier, now part of the assets, and it does earn a profit if properly run.

[2.3] Article 37 in Delegated Acts specifies CoC=6%.

However, this takes care of the first year only since SCR doesn't look further than that. In practice the contracts of a life insurance company last much longer, say up to K years ahead with similar solvency capital requirements $\mathrm{SCR}_1,\ldots,\mathrm{SCR}_{K-1}$ for each year up to the next to last one. Keeping capital to cover those has cost too, and a natural summary becomes

$$\mathrm{RM} = \mathrm{CoC} \times \sum_{k=0}^{K-1} \frac{\mathrm{SCR}_k}{(1+r_k)^{k+1}}, \tag{2.7}$$

where SCR_0=SCR. Note the discounting through the risk-free interest rate curve as before.

The Delegated Acts lay down that the risk margin should be calculated in this way 'in principle', but there is a practical problem in that SCR requires a lot of work even for year 0, as will emerge below. The Regulators have therefore opened for simplifications. One possibility is to calculate Best Estimates $\mathrm{BE}_1,\ldots,\mathrm{BE}_{K-1}$ similar to $\mathrm{BE}_0 = \mathrm{BE}$ at all future years on the basis of existing contracts (which is much simpler) and take

$$\mathrm{SCR}_k = \mathrm{SCR}_0 \times \frac{\mathrm{BE}_k}{\mathrm{BE}_0}, \qquad k = 0,\ldots,K-1, \tag{2.8}$$

assumimg that $\mathrm{SCR}_0,\ldots\mathrm{SRC}_{K-1}$ and $\mathrm{BE}_0,\ldots\mathrm{BE}_{K-1}$ are proportional (only an approximation). The risk margin can then be calculated from $\mathrm{SCR} = \mathrm{SCR}_0$ to which we now turn.

3 The solvency capital requirement.

3.1 The approach

The solvency capital requirement is not found by solving the equation (2.4); consult Section 4 to see why. Instead calculations in Solvency II proceeds through the representation in Figure 2 which is a simplified version of the operations of a life insurance company. At the top the company surplus (called BOF earlier) is influenced by its **Basic Operations** which consists of **investments** on the left and **insurance** proper on the right with their sources of risk listed under them at Layer 3. There are also two smaller components at Layer 1. **Operational** risk on the left is due to failure of internal administrative processes and control (which may create losses) and the **adjustment** term on the right captures situations when companies expect to be able to use the Technical Provisions to off-set losses[3.1]. Although operational risk *could* go sky-high, it is not expected to and the provisions for it is usually not very high. It is disregarded in the following along with the adjustment term so that only the investment and insurance modules are taken into account. There is on Layer 2 in the full Solvency II regime also an additional component due to the failure of debtors to honour their obligations. Such **credit** risk is also ignored.

All the variables in Figure 2 have been assigned solvency capital requirements or SCR's bearing their name, and that is how Solvency II operates. Starting at the bottom at Layer 3 the 99.5% percentiles of the various activities there are computed and then aggregated to similar percentiles one layer higher up. In Figure 2 this means that $\mathrm{SCR}^{\mathrm{mark}}$ for market (or investment) risk is derived from all the sub-variables under it and the same for $\mathrm{SCR}^{\mathrm{liab}}$ for liability (or insurance) risk. Subsequently $\mathrm{SCR}^{\mathrm{mark}}$ and $\mathrm{SCR}^{\mathrm{liab}}$ yields the company SCR at the top via the so-called Basic Solvency

[3.1] The mechanism is that companies had been expecting to transfer bonuses and rewards to their policy holders at its discretion so that it may withhold them when things go badly.

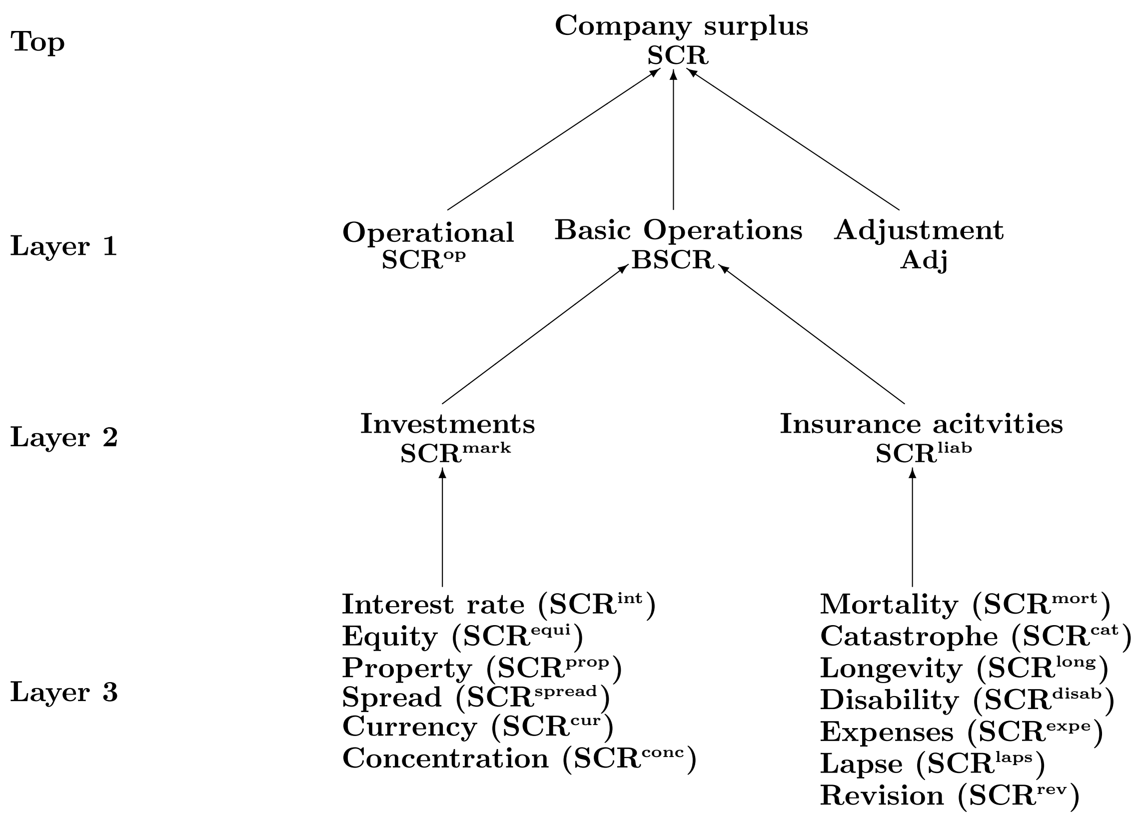

Figure 2 *A simplified Solvency II representation of life insurance risk.*

Capital Requirement (or **BSCR**) which is our simplified situation with no operation or adjustment risk coincides with SCR. How the aggregations are carried out comes next.

3.2 The standard formula

Procedure The accumulation issue in Section 3.1 can be expressed as follows. Consider one of the layers in Figure 2 with underlying risk variables $X_1, \ldots, X_n$ and 99.5% percentiles $\mathrm{SCR}_1, \ldots, \mathrm{SCR}_n$. Each X_i represents a loss (or a gain), and their sum

$$Y = X_1 + \ldots + X_n \tag{3.1}$$

is the total loss (or gain) of the layer itself. We seek its 99.5% percentile SCR_y. Independence between components would have been hopelessly unrealistic, and the standard model in Solvency II introduces correlations

$$\rho_{ij} = \mathrm{cor}(X_i, X_j) \tag{3.2}$$

which yield the aggregation rule

$$\mathrm{SCR}_y = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \times \mathrm{SCR}_i \times \mathrm{SCR}_j \right)^{1/2}, \tag{3.3}$$

known as the **Standard Formula**. It has the following property:

Theorem *The Standard Formula (3.3) is exact when $X_1, \ldots, X_n$ are Gaussian risks with mean* 0.

Proof Let ϕ (= 2.52) be the 99.5%-percentile of the standard normal distribution and write $\mathrm{SD}_i = \sqrt{\mathrm{var}(X_i)}$. Then

$$\mathrm{SCR}_i = \phi \times \mathrm{SD}_i, \quad i = 1, \ldots, n,$$

and if $\mathrm{SD}_y = \sqrt{\mathrm{var}(Y)}$, then by the ordinary variance/covariance formula for sums of random varaiables

$$\mathrm{SD}_y = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \times \mathrm{SD}_i \times \mathrm{SD}_j \right)^{1/2}.$$

But now (3.3) follows after multiplying with ϕ on both sides since $\mathrm{SCR}_y = \phi \times \mathrm{SD}$.

Discussion The tree in Figure 2 can now be climbed from bottom to top when the SCR's at Layer 3 are specified since the Delegated Acts specify correlations ρ_{ij} at all layers. We first obtain $\mathrm{SCR}^{\mathrm{market}}$ and $\mathrm{SCR}^{\mathrm{liab}}$ at Layer 2 and from those BSCR=SCR for the entire company. Operational risk and Adjustment risk at Layer 1 has been ignored, but had they been present, they could have been worked in as well. However, how plausible are the underlying conditions? Zero means $E(X_i) = 0$ appear unproblematic since the expected was taken out by the Best Estimate so that the remaining has expectation zero. The Gaussian assumption is more doubtfull, and is dependence between these variables always well described by correlations? The answer to the latter is a clear negative, yet it could at least lead to *conservative* evaluations which err on the prudent side; see also the closing Section 4.

An upper bound If $\rho_{ij} = 1$ are inserted in (3.3) for all (i, j), we arrive at the upper bound

$$\mathrm{SCR}_y \leq \mathrm{SCR}_1 + \ldots + \mathrm{SCR}_n. \tag{3.4}$$

This is is not true for non-Gaussian risks in general, but it does hold here[3.2] No use will be made of this in the present note, but there are many applications in other parts of the Solvency II regime.

Example At Layer 2 the correlation between market and libability risk is specified as $\rho_{12} = 0.25$ (see EIOPA (2014a), p.126) which means that their SCR's are aggregated to the company SCR=BSCR through

$$\mathrm{SCR} = \left((\mathrm{SCR}^{\mathrm{mark}})^2 + 0.5 \times \mathrm{SCR}^{\mathrm{mark}} \times \mathrm{SCR}^{\mathrm{mliab}} + (\mathrm{SCR}^{\mathrm{mliab}})^2 \right)^{1/2}. \tag{3.5}$$

3.3 Liability risk

Variables contributing to liability risk in life insurance are those on the right in Figure 2. The first five are discussed below whereas 'Lapse' (the sixth variable) due to losses from more clients leaving

[3.2] The argument runs as follows. Inserting all $\rho_{ij} = 1$ yields

$$\mathrm{SCR}_y \leq \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{SCR}_i \times \mathrm{SCR}_j \right)^{1/2} = \left(\sum_{i=1}^{n} \mathrm{SCR}_i \times \sum_{j=1}^{n} \mathrm{SCR}_j \right)^{1/2} = \sum_{i=1}^{n} \mathrm{SCR}_i.$$

the company than expected and 'Revision' (the seventh) caused by legal rules being changed, will be ignored. The correlations among the first five are specified in[3.3]. All the five SCR's needed are constructed from **Stress Tests**. This means that scenario shocks S (perceived as 99.5% percentiles) are offered by EIOPA and converted to losses by evaluating their impact on company portfolios.

How these calculations are carried out has a common structure that applies for all the remaining five variables except one. First the company must identify the contracts for which the shock makes the liabilities (the expected cost) go up. Suppose there are J of those with liabilities $L_1, \ldots, L_J$. Those must be recalculated under the stressed conditions which yield, say $L_1^s, \ldots, L_J^s$. The solvency capital requirement then becomes

$$\mathrm{SCR} = \left(\sum_{j=1}^{J} L_j^s - \mathrm{RE}^s \right) - \left(\sum_{j=1}^{J} L_j - \mathrm{RE} \right) \tag{3.6}$$

with RE and RE^s reinsurance compensations (under the standard and stressed conditions respectively) if the company has bought such protection. The calculation of $L_1, \ldots, L_J$ and $L_1^s, \ldots, L_J^s$ makes use of standard actuarial technique with the underlying models different for men and women.

Mortality ($\mathbf{SCR^{mort}}$) This is Solvency II jargon for adverse economic effects of mortality probabilities going up. Pension costs are not affected (they become smaller), but insurance against death is affected. If q_l is the probability of dying within one year at age l, then the shocked version is

$$q_l^s = (1+S) \times q_l \qquad \text{where} \qquad S = 15\%, \tag{3.7}$$

and Solvency II lais down that all contracts which become more expensive under q_l^s should be recalculated and inserted into (3.6).

Catastrophe ($\mathbf{SCR^{cat}}$) This signifies events such as pandemies or nuclear disasters leaving huge death in their wake. The stress model is now

$$q_l^s = q_l + S \qquad \text{where} \qquad S = 0.15\%. \tag{3.8}$$

The solvency capital requirement is calculated as explained for mortality risk. Contracts for which expected cost go up are singled out, the liabilities $L_1, \ldots, L_J$ and $L_1^s, \ldots, L_J^s$ are found and (3.6) yields the result after including reinsurance if necessary.

Longevity ($\mathbf{SCR^{long}}$) The shock is now towards lower mortalities; that's what longevity means, and the stress scenario becomes

$$q_l^s = (1-S) \times q_l \qquad \text{where} \qquad S = 20\%, \tag{3.9}$$

[3.3]The correlations between Mortality, Catastrophe, Longevity, Disability and Expense risk are offered in EIOPA (2014a), p.202. Their values are

	Mortality	Catastrophe	Longevity	Disability	Expenses
Mortality	1				
Catastrophe	0.25	1			
Longevity	-0.25	0.25	1		
Disability	0.25	0.25	0	1	
Expenses	0.25	0.25	0.25	0.50	1

and once again the company is required to identify the contracts for which expected cost goes up. In the present case those apply to pensions. Otherwise the procedure is the same; i.e separate calculations for men and women and the capital requirement defined by (3.6) after calculating the two sets of liabilities $L_1, \ldots, L_J$ and $L_1^s, \ldots, L_J^s$.

Disability ($\mathbf{SCR^{disab}}$) This module is in Solvency II called disability/morbidity with morbidity essentially the same as disability except for certain medical criteria which need not concern us. A disability scheme introduces two states a and i with a ('active') the normal, healthy one whereas individuals in state i ('disabled') benefit from a pension. Modelling requires two sets of probabilities $q_l(i|a)$ and $q_l(a|i)$ with the former the likelihood of becoming disabled at age l and the other of returning to 'active' (known as **rehabilitation**). The mathematical notation is as in Bølviken (2014). A complete model requires mortalities as well.

The Solvency II stress model for becoming disabled specifies one shock for the coming year (S_1) whereas another one (S_2) applies for all the subsequent years. In detail

$$q_l^{s_1}(i|a) = (1 + S_1) \times q_l(i|a) \qquad \text{where} \qquad S_1 = 35\% \tag{3.10}$$

for the first year and

$$q_l^{s_2}(i|a) = (1 + S_2) \times q_l(i|a) \qquad \text{where} \qquad S_2 = 25\% \tag{3.11}$$

for the second year and later. Then there is the rehabilitation stress which is

$$q_l(a|i) = (1 - S_3) \times q_l(a|i) \qquad \text{where} \qquad S_3 = 20\%, \tag{3.12}$$

a 20% downturn in the rehabilitation rate. As for the other modules the liabilities $L_1, \ldots, L_J$ for J policies in a disability portfolio must be recalculated under the stressed model. If $L_1^s, \ldots, L_J^s$ are the new, *higher* values, the solvency capital requirement $\mathrm{SCR^{disab}}$ now follows from (3.6).

Expenses ($\mathbf{SCR^{expe}}$) The cost of running a life insurance company is a part of the best estimate BE and has been assigned a certain value EI there ('EI' stands for Expenses Incurred). Recall that EI was based on a rate of inflation I which is now stressed as

$$I^{s_1} = I + S_1 \qquad \text{where} \qquad S_1 = 1\%, \tag{3.13}$$

and EI when recalculated under I^{s_1} increases to EI^{s_1}. There is also a second shock S_2 which applies to the level of the expenses, and the solvency capital requirement of this module becomes

$$\mathrm{SCR^{expe}} = (1 + S_2) \times \mathrm{EI}^{s_1} - \mathrm{EI} \qquad \text{where} \qquad S_2 = 10\%. \tag{3.14}$$

3.4 Market risk

Among the six sub-variables under the Market module in Figure 2 we shall ignore 'Currency' (the fifth) and 'Concentration' (the sixth)[3.4]. This is correct when all operations are in a single currency and the debtors of the company are sufficiently spread. What lies behind the remaining

[3.4] 'Currency' applies when a company has assets or liabilities abroad so that fluctutuations in exchange rates are causing risk (which may be considerable). 'Concentration' reflects how company credit distributes over counterparties and defines risk when exposures (in %) exceeds certain thresholds.

four contributions 'Interest rate', 'Equity', 'Property' and 'Spread' will be presented along with the quantifications of their solvency capital requirements. They are aggregated to $\mathrm{SCR}^{\mathrm{mark}}$ for the entire market module though the correlations in[3.5]. As in Section 3.3 the approach is through stresses everywhere.

Interest rate ($\mathbf{SCR}^{\mathrm{int}}$) The Best Estimate BE calculated from the liabilities $L_1, \dots, L_K$ in (2.6) go up when the risk-free rates of interests r_k go down. This is interest rate risk not to be confused with spread risk which applies to bonds and other fixed-income securities valued through their yield; see below. Although some of $L_1, \dots, L_k$ *could* go negative so that there are *increases* in the risk-free rates that create losses, it is for life insurance companies in practice the downwards shifts that are the threat, and risk caused by rising level (although included in Solvency II) is here ignored. The stressed version of the risk-free rates of interest is then

$$r_k^s = r_k \times (1 - S_k), \qquad k = 1, \dots, K \tag{3.15}$$

with $S_1, \dots, S_K$ coming from a shock curve specified in the Delegated Acts[3.6], and the solvency capital requirement becomes

$$\mathrm{SCR}^{\mathrm{int}} = \mathrm{BE}^s - \mathrm{BE}. \tag{3.16}$$

where BE^s is the value of the best estimate when r_k^s replaces r_k for all k in (2.6).

Equity ($\mathbf{SCR}^{\mathrm{equi}}$) Solvency II distinguishes between equity investments that are 'strategic' for the insurance company (Type 1) and those that are not (Type 2), and there are also special provisions for companies outside the European Economic Area (EEA) and the Organization for Economic Development (OECD). Ignoring the latter suppose A_1 and A_2 are the market values of equity of the two types. Then in the simplest case without options to protect investments

$$\mathrm{SCR}^{\mathrm{equi}} = (1 - S_1) \times A_1 + (1 - S_2) \times A_2 \tag{3.17}$$

where

$$\underset{\textit{strategic}}{S_1 = 22\%} \qquad \underset{\textit{not strategic}}{S_2 = 39\%.} \tag{3.18}$$

[3.5] Correlations for Interest rate, Equity, Property and Spread are offered on p. 138-139 in EIOPA (2014a). They are

	Interest rate	Equity	Property	Spread
Interest rate	1			
Equity	ρ	1		
Property	ρ	0.75	1	
Spread	ρ	0.75	0.50	1

where $\rho = 0$ if the interest rate risk of the company is due to rising levels and $\rho = 0.5$ otherwise. In this note $\rho = 0.5$.

[3.6] Article 167 presents the following table for the downwards shocks for the rate of interest:

k	≤ 1	2	3	4	5	6	7	8	9	10	12	15	90
S_k	0.75	0.65	0.56	0.50	0.46	042	0.39	0.36	0.33	0.31	0.29	0.27	0.20.

For missing k in the table you have to interpolate.

Here the shocks S_1 and S_2 represent downturns of the market and the stressed value of the stock becomes $A_j^s = S_j \times A_j$ for $j = 1, 2$ so that $A_j - A_j^s = (1 - S_j) \times A_j$ is the loss. In practice an insurance company may well have bought put options to limit the risk, and in such cases the computation of $\mathrm{SCR}^{\mathrm{equi}}$ become more complex since there is no longer a simple proportional link between the values before and after the shock.

Property ($\mathbf{SCR}^{\mathbf{prop}}$) Investment in property is one of the important instruments for insurance companies. The solvency capital requirement is simple and of the same form as in (3.17). If A is the value of the property portfolio, then

$$\mathrm{SCR}^{\mathrm{prop}} = (1 - S) \times A \qquad \text{where} \qquad S = 25\%, \tag{3.19}$$

ans the shock is thus a 25% decline of the market.

Spread ($\mathbf{SCR}^{\mathbf{spread}}$) Behind spread risk lies fixed-income securities where the right to a fixed cash flows $B_1, \ldots, B_K$ have been bought from an issuer. The traditional example is a bond, but instruments such as credit derivatives or securitized products have emerged throughout the last decades; some of which having been made almost infamous by the financial crisis of $2008 - 2009$. Solvency II has a quite complex and detailed treatment here, but we shall simplify and assume a traditional bond. There is a huge second-hand market for such securities and the value A of possessing the right to the cash flow $B_1, \ldots, B_K$ is determined by what investors are willing to pay. This is in turn influenced by expectations of future inflation and interest rates and above all by the perception of the danger of a default on the debt.

The **yield** y of such a security is the solution of the equation

$$A = \frac{B_1}{1+y} + \ldots + \frac{B_K}{(1+y)^K}, \tag{3.20}$$

and we also need the concept of **duration** which is

$$D = \sum_{k=1}^{K} k \times q_k \qquad \text{where} \qquad q_k = \frac{B_k/(1+y)^k}{A}. \tag{3.21}$$

Formally D is a mathematical expectation (since $q_1 + \ldots + q_K = 1$), and it is a measure of how long in time a cash flow is[3.7]. The **spread** of a bond is its yield minus the yield of very secure cash-flows of *the same duration* such as US treasury bonds or bonds issued by the European Central Bank.

Note that when the spread (and hence y) goes up, the value of the bond goes down. This is spread risk which depends on the duration of the bond and the credit quality of the issuer. The latter is linked to its ratings by the international rating bureaus. Solvency II has converted those into **credit quality steps** ω which varies from $\omega = 0$ (best) to $\omega = 6$ (worst) [3.8]. If the bond

[3.7] This is known as the **Maccaulay** version of duration. Solvency II uses the slightly different $D' = D/(1 + y/K)$.

[3.8] The relationships between the credit quality steps ω in Solvency II and the ratings published by Standard and & Poor (S&P), Fitch and Moodys are listed in Appendix MA in EIOIPA (2014b) as the table portfolio consists of J securities with market values $A_1, \dots, A_J$, ratings $\omega_1, \dots, \omega_J$ and durations $D_1, \dots, D_J$ the spread risk capital requirement becomes

$$\mathrm{SCR}^{\mathrm{spread}} = \sum_{j=1}^{J} (1 - S_j) \times A_j \qquad \text{where} \qquad S_j = 1 - b(\omega_j) \times D_j \tag{3.22}$$

On the right $b(\omega)$ are coefficients tabulated in Article 104 in the Delegated Acts[3.9]. The shock functions have a more general form $S(\omega, D)$ in Article 176, but the linear one in (3.22) is permitted.

4 Summing up.

The liabilities $L_1, \dots, L_K$ underlying the Best Estimate (2.4) lie at the heart of much Solvency II reporting and require actuarial technique to be evaluated. That you must pick up elsewhere. The regulatory regime has weaknesses. One of them is that the use of correlations to describe dependencies which has its limitations. There is, for example, a hedging effect in holding responsibility for both pension portfolios and insurance against death. If mortalities change, the loss on one part will be partially off-set by gain on the other, and this is not properly reflected in Solvency II specification[3.1]. Nor is risk reduction through so-called asset-liability management handled well. One of the strategies now is to adapt a fixed bond cash flow $B_1, \dots, B_K$ to given liabilities $L_1, \dots, L_K$ so that their profiles resemble each other with fluctuations in their difference going down[3.2].

For reasons such as those Solvency II opens for internal models developed by the companies themselves as replacements for the standard model treated in this note. Regulatory approval is necessary, and up to the time of writing (February, 2017) no insurance company in Norway has been permitted to use an internal model. This is different in the banking sector (where the analogy to Solvency II is Basel II) and in Denmark where the first supervisory approval of an internal model was given in October 2015.

Finally, it is in Solvency II documentation maintained that the regulation is so safe that there is over a one-year time horizon only 0.5% chance of a company running out of money. On the face of it there will then be 200 years between each case of company insolvency, or among two hundred companies one single default can be expected annually. There are several reasons for taking this with a grain of salt or at least regard it as unproven. Firstly, the equation (2.4) is not actually solved which is by the way a pretty hard problem as pointed out in Ohlsson and Lauzeningks (2009).

Solvency II	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$	$\omega = 6$
S&P/Fitch	AAA	AA	A	BBB	BB	bBB	bBB
Moody's	Aaa	Aa	A	Baa	Ba	B	B

[3.9] The coeffcients offered are

Credit quality	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$	$\omega = 6$	No rating
$b(\omega)$	0.9%	1.1%	1.4%	2.5%	4.5%	7.5%	7.5%	3.0%

[3.1] The correlation between mortality and longevity risk is specified as -0.25, often insuffieicnt to capture the reduction in risk.

[3.2] Such asset matching is difficult to achieve in Norway for the lack of bonds of suffciently long duration, but it is a popular strategy in Britain.

But even if we were able to crack that nut and build a huge model simulating all company operations and activities through techniques as in Daykin, Pentikäinen and Pesonen (1994) or Bølviken (2014), many of its parameters would be highly uncertain (and nor will they remain what they are for two centuries!). In other words, the projected probabilities of Basic Own Fund going negative would deviate from the true ones. One could also question the Gaussian assumption underlying the Standard Formula, but that might work better in life insurance than in general insurance where the studies in Savelli and Clemente (2011) and Alm (2015) reported solidity a good deal less than 99.5% in their examples. One of the reasons is that many variables in insurance are skewed to the right which makes the Gaussian distribution undersestimate risk; for an extension of the Standrad Formula to deal with this, consult Bølviken and Guillen (2016).

It follows that the criterion (2.4) should be seen more as a regulatory tool than as a scientifically founded statement, let us call it the vision of Solvency II, an ideal towards which the regulatory system is striving while trying to maintain simplicity and transparency in assumptions and methods.

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