Prøveeksamen

Problem 1 (Permanent disability insurance in discrete time) Let us have a look at a permanent disability insurance (permanent uførepensjonsforsikring). Let $x = 20$ years be the age of the insured at the beginning of the contract and let 40 be the age at maturity. Suppose that $\delta = 3\%$ (intensity rate) and that the yearly disability pension is given by 12000\$.

Assume the following transition rates

$$\mu_{*\diamondsuit}(t) = 0.0006, \mu_{*\dagger}(t) = 0.0015, \mu_{\diamondsuit\dagger}(t) = \mu_{*\dagger}(t).$$

Calculate the prospective reserve of the disability pension payments at time $t = 37$ given that $X_t = *$.

Problem 2 An insurance company issues a 10-year pure endowment (opplevelsesforsikring) to a life aged 55. Suppose that the endowment amount is given by 115000\$. Further, assume for this contract stochastic interest rates $r(t)$ described by the Vasicek model with parameters $r(0) = 0.035, a = 0.6, b = 0.04$ and $\sigma = 0.014$. Let $\lambda = -1$ (risk premium) and

$$\mu_{*\dagger}(t) = 0.015$$

be the constant transition rate.

Compute the prospective reserve of the endowment payment at time $t = 0$.

Hint: Use the formula for zero-coupon bond prices:

$$P(t,T) = \exp(-(T-t)R(T-t, r(t))),$$

where

$$\begin{aligned} & R(s,x) \\ = \; & (b - (\lambda\sigma)/a - \frac{\sigma^2}{2a^2}) - \frac{1}{a \cdot s}[((b - (\lambda\sigma)/a - \frac{\sigma^2}{2a^2}) - x)(1 - e^{-as}) - \frac{\sigma^2}{4a^2}(1 - e^{-as})^2]. \end{aligned}$$

Problem 3 An insurer offers a 10−year unit-linked term insurance (risikoforsikring med investeringsvalg) with a single premium $P = 10000$\$ to a life aged 45. An initial expense deduction of 4.5% is charged and the rest of the premium is invested in an equity fund whose dynamics S_t of its values over time is described by the Black-Scholes model with $S_0 = 1$. Further, management charges are deducted on a daily basis from the insured's account at a rate of $\beta = 0.7\%$ per year (i.e. in the sense of a continuous deduction based on the discount factor $e^{-\beta t}$). If death occurs during the contract period a death benefit of 120% of the fund value is provided.

Suppose that

(i)

$$\mu_{*\dagger}(t) = 0.0065.$$

(ii) Risk free rate of interest $r = 5\%$ per year, continuously compounded.
(iii) Volatility $\sigma = 27\%$ of S_t per year.

Calculate the prospective reserve of the benefit payments at time $t = 0$.

Problem 4 Again let us have a look at a disability income insurance with term two years issued to a healthy life aged 50. Premiums are paid continuously throughout the contract period while the policyholder is in the healthy state. Disability benefits are provided at the rate of 60000$ per year.

Assume that the following transition probabilities are applied to the insured:

$$\begin{aligned} p_{**}(50, 50+t) &= \frac{2}{3}\exp(-0.015t) + \frac{1}{3}\exp(-0.01t), \\ p_{*\dagger}(50, 50+t) &= 1 - \exp(-0.01t). \end{aligned}$$

Use an intensity rate of 5% to compute the annual premiums P.