Life Insurance and Finance (STK4500)

Exercises 2, 26.02.2026

Problem 1(Disability income insurance) A disability income insurance provides benefits during periods of sickness. There are no benefits after recovery. Typically, such a policy provides an annuity while the person is sick, whereas premiums are paid while the insured is healthy. On the other hand it could be also used for valuing lump sum payments in the case of sickness or death.

Assume in this model that the state of the insured $X_t \in S$ is described by a regular Markov chain with state space $S = \{*, \diamondsuit, \dagger\}$, where $*$ ="healthy", $\diamondsuit$ ="sick" and $\dagger$ ="dead". Suppose that the transition rates are given by the Gompertz-Makeham model as follows

$$\mu_{*\diamondsuit}(t) = a_1 + b_1 \exp(c_1 t), \mu_{\diamondsuit *}(t) = 0.1 \cdot \mu_{*\diamondsuit}(t), \mu_{*\dagger}(t) = a_2 + b_2 \exp(c_2 t), \mu_{\diamondsuit\dagger}(t) = \mu_{*\dagger}(t),$$

where $a_1 = 4 \cdot 10^{-4}, b_1 = 3.4674 \cdot 10^{-6}, c_1 = 0.138155$ and $a_2 = 5 \cdot 10^{-4}, b_2 = 7.5858 \cdot 10^{-5}, c_2 = 0.087498$.

Compute $p_{**}(x, x+10)$ and $p_{*\diamondsuit}(x, x+10)$ for $x = 60$ (years).

Problem 2 (Bootstrap method) A coupon bearing bond provides coupon payments $C_1, ..., C_m$ at future times $T_1 < ... < T_m$ and a principal at maturity T_m. Then using continuous compounding of interest rates the present value $B(t)$ of such a bond is given by the formula

$$B(t) = \sum_{i=1}^{m} C_i e^{-(T_i - t) \cdot R(t, T_i)} + X e^{-(T_m - t) \cdot R(t, T_m)}.$$

Consider coupon bonds given by the following 5 Treasury bonds:

$\underline{Bond Principal}$	$\underline{Time to maturity}$	$\underline{Annual coupon}$	$\underline{Bond price}$
(\$)	(years)	(\$)	(\$)
100	0.25	0	98.7
100	0.50	0	97.3
100	1.00	0	94.6
100	1.50	8	102.4
100	2.00	12	110.1

Suppose that the above coupons are semiannually paid .

Bootstrap the Treasury zero rates up to time-to-maturity of 2 years, that is find the initial term structure of interest rates by computing the rates $R(t, t+0.25), ..., R(t, t+2)$ based on the above data.

Problem 3 Assume an endowment policy with 250000 \$ death benefit and 125000 \$ survival benefit. The yearly premium (in continuous time) is given by 2200 \$. Suppose that the insured is 25 years at the beginning of the contract, which expires at the age of 65. Further, death occurs at 45 years. Let $\delta(s) = \log(1 + 0.03)$. Compute the prospective reserve $V^+(t, A)$ for $t = 30$ years.