Life Insurance and Finance (STK4500)

Exercises 3, 05.03.2026

Problem 1(Interest rates in a periodic economy) Consider interest rates in a "periodic economy" with a period of 8 states. Let us assume that such interest rates are modeled by a (homogeneous) Markov chain $i_t, t = 0, 1, 2, ...$ with the following transition probabilities for the corresponding states:

State		Interest rate (%)	p_{ii}	p_{ii+1}	p_{ii+2}
0	Start	5.0	0.1	0.7	0.2
1	Increasing interest	5.5	0.1	0.7	0.2
2	Max. interest	6.0	0.1	0.7	0.2
3	Decreasing interest	5.5	0.1	0.7	0.2
4	Average	5.0	0.1	0.7	0.2
5	Decreasing interest	4.5	0.1	0.7	0.2
6	Min. interest	4.0	0.1	0.7	0.2
7	Increasing interest	4.5	0.1	0.7	0.2

Calculate the probability that i_t is in state 2, given that i_0 is in state zero for $t = 4$.

Problem 2 Consider a disability insurance. Suppose for this model that there is no waiting period and that the disability pension is given by the fixed amount of 20000 \$ per year until the age of 65 years. Assume that annual premiums are 2500 \$ until the age of 65 years and that the age of the insured at the beginning of the contact is $x_0 = 30$ years.

Determine the policy functions $a_i(t)$ (generalized pension payments) and $a_{ij}(t)$ (generalized capital benefits).

Problem 3 Let us assume in Problem 2 a permanent disability model with transition rates given by

$$\mu_{*\diamondsuit}(t) = 0.0279, \mu_{*\dagger}(t) = 0.0229, \mu_{\diamondsuit\dagger}(t) = \mu_{*\dagger}(t).$$

Further , let the (constant) force of interest $\delta(t)$ be 3%.

Calculate the total reserve $V_j(t, A)$ for all states j with respect to $t = 60$ years and the time interval $T = [60, 65]$.