STK4500
Exercises 3

Prob. 1 :

$$P \overset{\text{def}}{=} \begin{pmatrix} 0.1 & 0.7 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.7 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.7 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0.2 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.7 \\ 0.7 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{pmatrix}$$

Chapman-Kolmogorov equation (see also Obl.ig. 1, Prob. 1)

$$P(i_4 = 2 \mid i_0 = 0) = \widehat{P}_{02}, \text{ where}$$

$$\widehat{P} = (\widehat{P}_{ij})_{0 \le i,j \le 7} \quad \text{with } \widehat{P} \overset{\text{def}}{=} P^4 = P \cdot P \cdot P \cdot P$$

$$\Rightarrow \quad \widehat{P}_{02} = 3.02\,\%$$

Prob. 2 : policy functions :

$$a_*(t) = \begin{cases} 0, & \text{if } 0 \le t < 30 \\ (-2500)(t-30), & \text{if } t \in [30, 65) \\ (-2500)(65-30), & \text{if } t \ge 65 \end{cases}$$

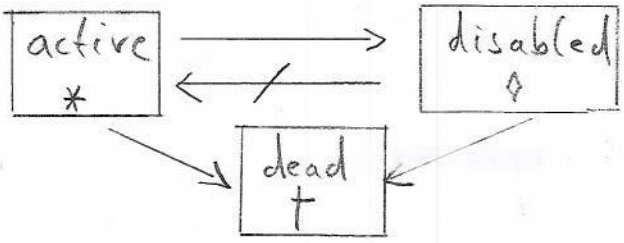
← premium payments

$$a_\emptyset(t) = \begin{cases} 0, & \text{if } 0 \le t < 30 \\ 20000 \cdot (t-30), & \text{if } t \in [30, 65) \\ 20000\,(65-30), & \text{if } t \ge 65 \end{cases}$$

← disability pension payments

Exerc 3

Prob. 3:



$V_*^+(60)$, $V_\diamond^+(60)$?

idea: Explicit formula for the prospective reserves (Th. 6.6):

$$V_j^+(t) = \frac{1}{v(t)} \left\{ \sum_{g \in S} \int_{(t,\infty)} v(s)\, p_{jg}(t,s)\, da_g(s) + \sum_{g \in S} \int_{(t,\infty)} v(s)\, p_{jg}(t,s) \Big(\sum_{\substack{h \in S_1 \\ h \neq g}} \underbrace{a_{gh}(s)}_{=0} \mu_{gh}(s) \Big) ds \right\} = 0 \qquad (*)$$

Here: $a_*(t) = \begin{cases} 0, & t < 30 \\ (t-30)\cdot 2500, & t \in [30,65) \\ -(65-30)2500, & t \geq 65 \end{cases}$

and $a_\diamond(t) = \begin{cases} 0, & t < 30 \\ t \cdot 20000, & t \in [30,65) \\ (65-30)\cdot 20000, & t \geq 65 \end{cases}$

$$\Rightarrow a_*(t) = \int_0^t (-2500)\, \mathbb{1}^{(s)}_{[30,65)}\, ds, \quad a_\diamond(t) = \int_0^t 20000 \cdot \mathbb{1}^{(s)}_{[30,65)}\, ds$$

($\frac{d}{dt} a_*(s)$, $\frac{d}{dt} a_\diamond(s)$)

Recall: $A(s) = \int_0^s g(u)\, du \Rightarrow \int f(s)\, dA(s) = \int f(s) \cdot g(s)\, ds$

$$\overset{(*)}{\Rightarrow} V_*^+(60) \overset{a_\dagger(s)=0}{=} \frac{1}{v(60)} \left\{ \int_{(60,\infty)} v(s)\, p_{**}(60,s)\, da_*(s) + \int_{(60,\infty)} v(s)\, p_{*\diamond}(60,s)\, da_\diamond(s) \right\}$$

$$= \frac{1}{v(60)} \left\{ \int_{(60,65)} v(s)\, \underbrace{p_{**}(60,s)}_{\text{Th. 3.17}} (-2500)\, ds + \int_{(60,65)} v(s)\, p_{*\diamond}(60,s) \cdot 20000\, ds \right\}$$

$$\text{Th. 3.17} \longrightarrow p_{**}(60,s) = \exp\left(-\int_{60}^{s} \mu_{*\diamond}(u) + \mu_{*\dagger}(u)\, du \right)$$

Exerc. 1, Prob. 4: $p_{*\diamond}(60,s) = \int_{60}^{s} p_{**}(60,t)\, \underbrace{\mu_{*\diamond}(t)}_{=0.0279} \exp\left(-\int_t^s \underbrace{\mu_{\diamond\dagger}(u)}_{=0.0229}\, du \right) dt$

$$\longrightarrow V_*^+(60) = -5193.918\,\$$$

Since $p_{\diamond *}(s,t) = 0$, we also get

$$V_\diamond^+(60) = \frac{1}{v(60)} \int_{(60,65]} v(s)\, \underbrace{p_{\diamond\diamond}(60,s)}_{= p_{\diamond\diamond}(60,s)} \cdot 20000\, \mathbb{1}^{(s)}_{[30,65)}\, ds$$

$$= 87962.7\,\$$$