STK 4500

Exercises 4

Prob. 1: alive (*) → dead (†)

annual premiums P and $V^{\dagger}_{*}(t)$, $t = 0, 1, \dots, 10$?

idea: Thiele's difference equation (Th. 6.14') and Remark 6.15

$$a^{Pre}_{*}(t) = \begin{cases} -P, & \text{if } 50+t < 60 \\ 0 & \text{else} \end{cases}$$

$$a^{Post}_{*\dagger}(t) = \begin{cases} 200000, & \text{if } 50+t < 60 \\ 0 & \text{else} \end{cases}$$

Thiele's diff. eq. $\Rightarrow$

$$V^{\dagger}_{*}(t) = a^{Pre}_{*}(t) + e^{-\delta} p_{*\dagger}(t,t+1) \left\{ a^{Post}_{*\dagger}(t) + \underbrace{V^{\dagger}_{\dagger}(t+1)}_{=0} \right\} + e^{-\delta} p_{**}(t,t+1) V^{\dagger}_{*}(t+1)$$

($e^{-\delta} p_{*\dagger}(t,t+1) = V_t$)

We know that

$$V^{\dagger}_{*}(t) \overset{\text{def.}}{=} V_*(t, A^{Pre}_{*}) + V_*(t, A^{Post}_{*\dagger}) \qquad (*)$$

$V_*(t, A^{Pre}_{*})$ ← premium income; $V_*(t, A^{Post}_{*\dagger})$ ← reserve for benefits

On the other hand

$$V_*(t, A^{Pre}_{*}) = P \cdot V_*(t, \tilde{A}^{Pre}_{*}), \text{ where } \tilde{a}^{Pre}_{*}(t) = \begin{cases} -1, & \text{if } 50+t < 60 \\ 0 & \text{else} \end{cases}$$

$\Rightarrow$ 2 steps:

1. Calculation of $V_*(t, A^{Post}_{*\dagger})$:

Thiele's diff. eq. for $a^{Pre}_{*}(t) = 0$ $\Rightarrow$

$$V_*(t, A^{Post}_{*\dagger}) = e^{-\delta} p_{*\dagger}(t,t+1) \underbrace{a^{Post}_{*\dagger}(t)}_{t+1} + e^{-\delta} p_{**}(t,t+1) V^{\dagger}_{*}(t+1, A^{Post}_{*\dagger})$$

$$p_{**}(t,t+1) = \overline{p}_{**}(t,t+1) = \exp\left(-\int_t^{t+1} \mu_{*\dagger}(50+u)\, du \right)$$

$$= \exp(-0.00225 - 0.0005 \cdot t)$$

$$p_{*\dagger}(t,t+1) = 1 - p_{**}(t,t+1)$$

$V^{\dagger}_{*}(10, A^{Post}_{*\dagger}) = 0 \Rightarrow$

$$V^{\dagger}_{*}(9, A^{Post}_{*\dagger}) = 1312.23, \; V^{\dagger}_{*}(8, A^{Post}_{*\dagger}) = 2487.2, \dots,$$

$$V^{\dagger}_{*}(0, A^{Post}_{*\dagger}) = 7520.29$$

2. Calculation of $V_*(t, \tilde{A}^{Pre}_{*})$:

Thiele's diff. eq. for $P = 1$ and $a^{Post}_{*\dagger}(t) = 0$ $\Rightarrow$

$$V^{\dagger}_{*}(t, \tilde{A}^{Pre}_{*}) = \tilde{a}^{Pre}_{*}(t) + e^{-\delta} p_{**}(t,t+1) V^{\dagger}_{*}(t+1, \tilde{A}^{Pre}_{*})$$

$V^{\dagger}_{*}(10, \tilde{A}^{Pre}_{*}) = 0 \Rightarrow$

$$V^{\dagger}_{*}(9, \tilde{A}^{Pre}_{*}) = -1, \; V^{\dagger}_{*}(8, \tilde{A}^{Pre}_{*}) = -1.96923, \dots,$$

$$V^{\dagger}_{*}(0, \tilde{A}^{Pre}_{*}) = -8.82407$$

$V^{\dagger}_{*}(0) = 0$ ← equivalence principle

$(*) \Rightarrow$

$$P = V^{\dagger}_{*}(0, A^{Post}_{*\dagger}) \,/\, V^{\dagger}_{*}(0, \tilde{A}^{Pre}_{*}) = 852.247\,\$$$

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Prob. 2 : disability insurance;

We know from Exerc. 1, Prob. 4 that

$$p_{*\emptyset}(s,T) = \int_s^T p_{**}(s,t)\,\mu_{*\emptyset}(t)\exp\left(-\int_t^T \mu_{\emptyset\dagger}(u)\,du\right)dt\,,$$

where $$p_{**}(s,t) = \exp\left(-\int_s^t \{\mu_{*\dagger}(u)+\mu_{*\emptyset}(u)\}\,du\right)$$

Further, $$p_{\emptyset\emptyset}(s,t) = \exp\left(-\int_s^t \mu_{\emptyset\dagger}(u)\,du\right)$$

$\longrightarrow$ $p_{*\dagger}(s,t) = 1 - p_{**}(s,t) - p_{*\emptyset}(s,t)$, $p_{\emptyset\dagger}(s,t) = 1 - p_{\emptyset\emptyset}(s,t)$

policy function: $a_\emptyset^{pre}(t) = \begin{cases} 12000, & \text{if } t\in[45,65) \\ 0 & \text{else} \end{cases}$ ← disability pension payment

$\xrightarrow{\text{Thiele's difference eq.}}$ 2-dim. recursion:

$$V_*^\dagger(t) = e^{-\delta}p_{**}(t,t+1)V_*^\dagger(t+1) + e^{-\delta}p_{*\emptyset}(t,t+1)V_\emptyset^\dagger(t+1)$$

and

$$V_\emptyset^\dagger(t) = a_\emptyset^{pre}(t) + e^{-\delta}p_{\emptyset\emptyset}(t,t+1)V_\emptyset^\dagger(t+1)$$

with terminal values $V_*^\dagger(65) = V_\emptyset(65) = 0$

$\longrightarrow$ (ii) $V_*^\dagger(65)=0$, $V_*^\dagger(64)=0$, $V_*^\dagger(63)=260.278$, $V_*^\dagger(62)=692.136$, ..., $V_*^\dagger(47)=7266.52$, $V_*^\dagger(46)=7406.6$, $V_*^\dagger(45)=7516.62$ ♯

check these numbers!!! $V_\emptyset^\dagger(65)=0$, $V_\emptyset^\dagger(64)=12000$, $V_\emptyset^\dagger(63)=23413.2$, $V_\emptyset^\dagger(62)=34304.8$, ...,
$V_\emptyset^\dagger(47)=158848$, $V_\emptyset^\dagger(46)=165394$ ($V_\emptyset^\dagger(45)=171774$)

and (i) $V_*^\dagger(65)=0$, $V_*^\dagger(64)=0$, $V_*^\dagger(63)=317.549$, $V_*^\dagger(62)=919.925$, ...,
$V_*^\dagger(47)=28320.5$, $V_*^\dagger(46)=30646$, $V_*^\dagger(45)=32974.3$ ♯

$V_\emptyset^\dagger(65)=0$, $V_\emptyset^\dagger(64)=12000$, $V_\emptyset^\dagger(63)=23381.7$, $V_\emptyset^\dagger(62)=34177$, ...,
$V_\emptyset^\dagger(47)=143023$, $V_\emptyset^\dagger(46)=147654$ ($V_\emptyset^\dagger(45)=152046$)