16th March, 2026

STK4500/9500

Mandatory assignment 1 of 1

Submission deadline

Thursday 9th APRIL 2026, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have one attempt to pass the assignment. This means that there are no second attempts.

For courses on bachelor level, you can choose between scanning handwritten notes or using a typesetting software for mathematics (e.g. LaTeX). Scanned pages must be clearly legible. For courses on master level the assignment must be written with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. Remember to include any relevant programming code and resulting plots and figures, in the PDF-file.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

CHOOSE 3 PROBLEMS FROM THE FOLLOWING LIST.

Problem 1. Suppose that X_n describes the transition of employees in an insurance company at the end of every six months between the three states

State 0- junior management

State 1- senior management

State 2- terminated

Assume that X_n,$n \geq 0$ is modeled by a homogeneous Markov chain with transition matrix P given by

$$P = \begin{pmatrix} 0.84 & 0.10 & 0.06 \\ 0.00 & 0.93 & 0.07 \\ 0.00 & 0.00 & 1.00 \end{pmatrix}.$$

Compute the probability that an employee who is at 1st of January 2019 in the junior management will be terminated at the end of 2019.

Problem 2. Let us have a look at an accidental death model with state 0: alive, state 1: dead- accident and state 2: dead- other causes. Suppose that the transition rates are given by

$$\mu_{01}(t) = 10^{-5} \text{ and } \mu_{02}(t) = A + Bc^t$$

for $A = 5 \cdot 10^{-5}$, $B = 7.6 \cdot 10^{-5}$ and $c = 1.09$.

Using this model, the insurance company wants to calculate premiums for a special 10-year term life insurance policy. The basic sum insured is 175000$. If death occurs as a result of an accident the death benefit is 270000$. The death benefits are paid at the end of the year of death. The constant yearly premiums are paid during the contract period. Assume that the intensity rate is constant and given by $\delta = 3.5\%$. The insured enters the policy at the age of 30.

Determine the annual premium for this policy by using the equivalence principle and calculate the prospective reserve $V_0^+(35)$.

Problem 3. Consider a permanent disability model with the states 0 ("healthy"), 1 ("permanently disabled") and 2 ("dead"). An insurer applies this model to the pricing of an insurance contract with term 2 years. The policy, which is issued to a life aged 58 years, pays a benefit of 110000$ in the case of death from the healthy state. Further, the benefit of 80000$ is provided, if permanent disability occurs, and 30000$ in the case of death

after permanent disability. The constant premiums are payable at the beginning of each policy year.

Assume the transition probabilities

x years	$p_{00}(x, x+1)$	$p_{01}(x, x+1)$	$p_{02}(x, x+1)$	$p_{11}(x, x+1)$	$p_{12}(x, x+1)$
58	0.995	0.002	0.003	0.992	0.008
59	0.993	0.003	0.004	0.990	0.010

Compute the annual premium for the effective interest rate $\delta = 3\%$.

Problem 4. An endowment is issued to a life aged 42 with 67 years as age at maturity. Let $\delta = 2.5\%$ be the constant annual intensity rate and assume that the death benefit is 280000\$. Suppose the survival benefit is given by 140000\$.

Further, assume that the mortality rates are described by the generalized Gompertz-Makeham law as follows:

$$\mu_{*\dagger}(t) = \exp(-7.85785 + 0.01538 \cdot t + 5.77355 \cdot 10^{-4} \cdot t^2).$$

Determine the (constant) annual premiums P based on the equivalence principle and compute the prospective reserves.

Problem 5. Consider a 10-year term insurance. The policy is issued to a life aged 50. The death benefit of 220000\$ is paid at the instant of death and premiums are continuously paid at a constant rate throughout the contract period. Further, no benefit is provided, if the policyholder lapses, that is cancels the policy during the contract period.

The transition rates with respect to the states 0 ("active"), 1 ("dead") and 2 ("lapsed") are given by

$$\mu_{01}(t) = 0.002 + 0.0005 \cdot (t - 50) \text{ and } \mu_{02}(t) = 0.05.$$

Compute the annual premium rate in the case of a force of interest $\delta = 3.5\%$.

Problem 6. Assume a pension on two lives with ages $x_1 = 36$ and $x_2 = 41$ at the beginning of the policy. Suppose that the two insured die independently with the same mortality rates as in Problem 4. Hence $**.*\dagger, \dagger*$ and $\dagger\dagger$ are the possible states of the two lives. Let us also assume that the pension payments are made according to

$$a_{**}^{\text{Pre}}(t) = \begin{cases} 0 & \text{if } 36 + t < 65 \\ 15000\$ & \text{else} \end{cases}, a_{*\dagger}^{\text{Pre}}(t) = a_{\dagger*}^{\text{Pre}}(t) = \begin{cases} 0 & \text{if } 36 + t < 65 \\ 11000\$ & \text{else} \end{cases}.$$

Similarly annual premiums P are payable. The annual intensity rate is 3.5% and 114 the maximal possible age of each person.

Compute P by using the equivalence principle.

Problem 7. (i) Consider as in Exercises 3, Problem 1 interest rates in a "periodic economy" with a period of 8 states. The interest rates in this model are described by a (homogeneous) Markov chain $i_t, t = 0, 1, 2, ...$ with the following transition probabilities for the corresponding states:

State		Interest rate (%)	p_{ii}	p_{ii+1}	p_{ii+2}
0	Start	5.0	0.1	0.7	0.2
1	Increasing interest	5.5	0.1	0.7	0.2
2	Max. interest	6.0	0.1	0.7	0.2
3	Decreasing interest	5.5	0.1	0.7	0.2
4	Average	5.0	0.1	0.7	0.2
5	Decreasing interest	4.5	0.1	0.7	0.2
6	Min. interest	4.0	0.1	0.7	0.2
7	Increasing interest	4.5	0.1	0.7	0.2

Simulate 10 or more paths of $i_t, t = 0, 1, 2, ..., 10$.

(ii) Suppose that the dynamics of overnight rates $r(t)$ is described by the Vasicek model

$$dr(t) = a(b - r(t)dt + \sigma dB_t,$$

where $a \geq 0$ is the mean reversion, $b \geq 0$ the reversion level and $\sigma \geq 0$ the volatility of the rates.

Denote by r_i the overnight rate at the end of the ith interval of constant length δ (in years), $i = 0, 1, ..., n$. Consider the following data set of Canadian overnight money market financing rates during 9 consecutive days:

$\underline{Day}$	$\underline{r_i\%}$	$\underline{Day}$	$\underline{r_i\%}$
01/06/07	4.2647	11/06/07	4.2438
04/06/07	4.2622	12/06/07	4.2481
05/06/07	4.2536	13/06/07	4.2412
06/06/07	4.2433		
07/06/07	4.2452		
08/06/07	4.2423		

Suppose that $\delta = \frac{1}{252}$ (i.e. 252 trading days per year).

Compute the maximum likelihood estimators for a, b and σ and apply these estimators to simulate 10 or more paths of $r(t), 0 \leq t \leq 1$ (1 year) by

using the recursion (discrete time version of the Vasicek model):

$$r_n = r_{n-1} + a(b - r_{n-1}) \cdot \delta + \sigma \cdot \sqrt{\delta} \cdot g_n, n \geq 0$$

with initial value $r_0 = 4.2647$, where $g_j, j \geq 0$ are i.i.d. standard normal random variables.

Problem 8. Assume a Markov chain $X_n, n \geq 0$ which is only observed at certain times. In the first instance suppose that J is some subset of the state space I and that we observe the chain only when it takes values in J. For example, in Problem 7 (i) we may think of a bond market trader who only wants to deal with interest rates in a certain "region" J. The resulting chain $Y_m, m \geq 0$ may be obtained formally by setting

$$Y_m = X_{T_m},$$

where

$$T_0 \stackrel{def}{=} \inf \{n \geq 0 : X_n \in J\}$$

and, for $m = 0, 1, 2, ...$

$$T_{m+1} \stackrel{def}{=} \inf \{n > T_m : X_n \in J\}.$$

Assume that

$$P(T_m < \infty) = 1$$

for all m.

(i) Show that $Y_m, m \geq 0$ is a Markov chain with state space J, that is

$$\begin{aligned} & P\left(Y_{m+1} = i_{m+1} \left| Y_0 = i_0, ..., Y_m = i_m \right.\right) \\ = \; & P\left(Y_{m+1} = i_{m+1} \left| Y_m = i_m \right.\right) \end{aligned}$$

for all $i_0, ..., i_{m+1} \in J$.

(ii) Prove that the transition probabilities of $Y_m, m \geq 0$, denoted by $\overline{p}_{i,j}$, are given by

$$\overline{p}_{i,j} = P\left(X_{T_1} = j \left| X_0 = i \right.\right), i, j \in J.$$

(iii) Show that

$$\overline{p}_{i,j} = h_i^j,$$

where for $j \in J$ the vector $\left(h_i^j\right)_{i \in J}$ is the minimal non-negative solution to the linear equations

$$h_i^j = p_{ij} + \sum_{k \notin J} p_{ik} h_k^j.$$