

STK4540: Non-life Insurance Mathematics

First assignment

To be submitted on Canvas no later than 8 October 2026, 2pm.

Exercise 1

Recall that a random variable θ is *gamma* distributed with *shape* parameter $\alpha > 0$ and *rate* parameter $\beta > 0$ if it has a density function f_θ given by

$$f_\theta(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0,$$

where

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

denotes the *gamma* function. We write $\theta \sim \Gamma(\alpha, \beta)$.

Let $\tilde{N}$ be a standard homogeneous Poisson process and let $\mu : [0, \infty) \rightarrow [0, \infty)$ be the mean value function of a Poisson process on $[0, \infty)$. Let $\theta \sim \Gamma(\alpha, \beta)$ (independent of $\tilde{N}$) and define the stochastic process

$$N(t) = \tilde{N}(\theta\mu(t)), \quad t \geq 0.$$

Prove that the distribution of the *mixed* Poisson process N with mixing variable θ is *negative binomial* with parameters $(p, b) = \left(\frac{\beta}{\beta + \mu(t)}, \alpha\right)$.

Exercise 2

An insurance company sells policies to protect private households against water claims.

Suppose that we are given the following claim count data of ten years:

t	1	2	3	4	5	6	7	8	9	10
$N(t)$	1000	997	985	989	1056	1070	994	986	1093	1054
$v(t)$	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000

Table 1: Observed claims counts $N(t)$ and corresponding policy volumes $v(t)$.



FIGURE 1: A house full of water

The aim of this exercise is to find a reasonable model for N and estimate the parameters involved in the model using a classical frequentist approach.

- (a) Assume that N is Poisson distributed. It makes sense to assume that the number of claims has an intensity proportional to the volume of policies sold by the company. For instance, if $v(t)$ denotes the number of active policies by time t , it makes sense to assume that $N(t)$ is Poisson distributed with intensity function $\mu(t) = \lambda v(t)$ for some suitable $\lambda > 0$. Estimate the claims frequency parameter $\lambda > 0$ and calculate an estimated, roughly 70%-confidence interval for λ . What do you observe?
- (b) Perform a χ^2 -goodness-of-fit hypothesis test at the significance level of 5% to test the null hypothesis of having Poisson distributions. Hint: try to find a reasonable test-statistic and argue that its distribution can roughly be approximated by a χ^2 -distribution.
- (c) Estimate the claims frequency parameter $\lambda > 0$ and the dispersion parameter $\gamma > 0$ when N is a mixed Poisson process with mixing variable $\theta \sim \Gamma(\gamma, \gamma)$ and intensity function $\mu(t) = \lambda v(t)$ and calculate an estimated, roughly 70%-confidence interval for λ . What do you observe?

Exercise 3

Consider an insurance policy that protects a personal belonging. This insurance has two possible coverage amounts. If the damage is severe, it pays \$1000, otherwise it pays \$500 with probabilities

$$P(X = 500) = 0.8, \quad P(X = 1000) = 0.2$$

If we assume that the claim number process follows a standard homogeneous Poisson distribution and the time is measured in years, compute the probability that the claim amount after one year does not exceed \$1500.

Approximate the distribution of S for the 5 first years by using a normal approximation and Panjer recursion scheme and compare the two approximations. Comment.