

Solutions theoretical exercises for STK4900/9900.

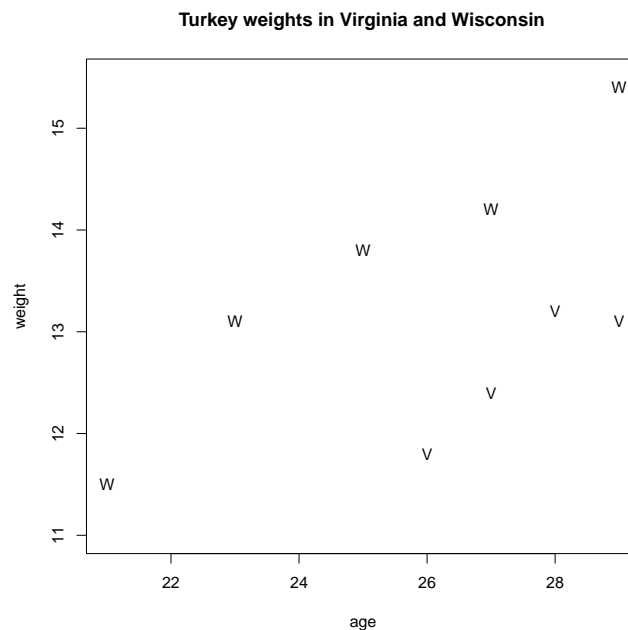
Exercise 6

- a) The average turkey weight in Virginia is $\bar{Y}_V = 12.625$ with a standard deviation $s_V = 0.665$. Correspondingly the average weight in Wisconsin equals $\bar{Y}_W = 13.6$ with a standard deviation $s_W = 1.44$.

We find the 95% confidence interval for difference in mean weight in the two states as $\bar{Y}_W - \bar{Y}_V \pm c \cdot se(\bar{Y}_W - \bar{Y}_V)$ where (1) $se(\bar{Y}_W - \bar{Y}_V) = s_p \sqrt{1/n_W + 1/n_V}$ (assuming that variances are equal in the two states) and the pooled standard deviation is calculated by $s_p^2 = ((n_W - 1)s_W^2 + (n_V - 1)s_V^2)/(n_W + n_V - 2) = 1.17^2$ since $n_W = 5$ and $n_V = 4$ and (2) c is the 97.5% percentile in a t_7 distribution = 2.365.

Thus the interval becomes (-0.88, 2.83). Since zero lies within the interval we do not reject a null hypothesis about the means in the states being equal.

- b) Below the turkey weights are plotted against age and state is indicated by "V" or "W":



For both states the weights appear to increase linearly with age and the slope of the two regression lines could well be equal.

- c) According to the plot a good model may be a multiple linear regression with covariates x_{1i} as the weights and x_{i2} being an indicator of

Wisconsin. The model for weight is then $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$. The β_1 is the increase in weight per week and β_2 the difference between Wisconsin and Virginia for two equally old turkeys.

- d) The 95% confidence interval for β_2 is given as $\hat{\beta}_2 \pm c \cdot se(\hat{\beta}_2)$ where $c = 2.447$ is the 97.5 percentile of a t-distribution with 6 degrees of freedom. The interval is calculated to be (1.51, 2.66) and since the interval does not include zero we conclude that weights in Wisconsin and Virginia indeed differ significantly when we have taken age into account.

Age is thus a confounding variable in such a way that a real difference will not be recognized if we do not take account of the variables.