What you must know

This document collects and expands the remarks "What you should have learned in this section", which you can find at the end of the sections in the compendium, and which summarize the most important theory and what kind of exercises you should be able to solve.

Part I

Section 1.2

The inner product which we use for function spaces. Definition of the Fourier spaces, and the orthogonality of the Fourier basis. Fourier series approximations as best approximations. Formulas for the Fourier coefficients. Using Matlab to plot Fourier series. For symmetric /antisymmetric functions, Fourier series are actually cosine/sine series.

Section 1.3

The complex Fourier basis and its orthonormality.

Section 1.4

The convergence rate of a Fourier series depends on the regularity of the function. How this motivates the symmetric extension of a function. Some simple Fourier series pairs. Certain properties of Fourier series, for instance how delay of a function or multiplication with a complex exponential affect the Fourier coefficients.

Section 2.1

Matlab operations for reading, writing, and listening to sound. Construct sounds such as pure tones, and the square and triangle waves, from mathematical formulas. Comparing a sound with its Fourier series. Changing the sample rate, adding noise, or playing a sound backwards.

Section 2.2

The definition of the Fourier basis and its orthonormality. The definition of the Discrete Fourier Transfrom as a change of coordinates to the Fourier basis, its inverse, and its unitarity. How to apply the DFT to a sum of sinusoids. Properties of the DFT, such as conjugate symmetry when the vector is real, how it treats delayed vectors, or vectors multiplied with a complex exponential.

Section 2.3

Translation between DFT index and frequency. In particular DFT indices for high and low frequencies. How one can use the DFT to adjust frequencies in sound.

Section 2.4

How the FFT algorithm works by splitting into two FFT's of half the length. Simple FFT implementation. Reduction in the number of arithmetic operations with the FFT.

Section 3.1

How to write down the circulant Toeplitz matrix from a digital filter expression, and vice versa. How to find the first column of this matrix ($\boldsymbol{s}$) from the filter coefficients ($\boldsymbol{t}$), and vice versa.

Section 3.2

The formal definition of a digital filter in terms of having the Fourier vectors as eigenvectors. The definition of the vector frequency response in terms of the corresponding eigenvalues. Digital filters are circulant Toeplitz matrices. For filters, eigenvalues can be computed by taking the DFT of the first column $\boldsymbol{s}$, and there is no need to compute eigenvectors explicitly. The definition of time-invariance. You should know all the equivalent characterizations of a filter. How to apply a digital filter to a sum of sines or cosines, by splitting these into a sum of eigenvectors.

Section 3.3

The definition of the continuous frequency response in terms of the filter coefficients $\boldsymbol{t}$. Connection with the vector frequency response. Properties of the continuous frequency response, in particular that the product of two frequency responses equals the frequency response of the product. How to compute the frequency response of the product of two filters, and how to find the filter coefficients when the continuous frequency response is known.

Section 3.4

The compact filter notation for filters with a finite number of filter coefficients. The definition of convolution, its connection with filters, and the Matlab `conv`-operation for computing convolution. Connection between applying a filter and multiplying polynomials.

Section 3.5

Simple examples of filters, such as time delay filters and filters which add echo. Lowpass and highpass filters and their frequency responses, and their interpretation as treble- and bass-reducing filters. Moving average filters, and filters arising from rows in Pascal's triangle, as examples of such filters. How to pass between lowpass and highpass filters by adding an alternating sign to the filter cofficients.

Part II

Section 5.2

Definition of resolution spaces (V_m), detail spaces (W_m), scaling function (ϕ), and mother wavelet (ψ) for the wavelet based on piecewise constant functions. The nesting of resultion spaces, and how one can project from one resolution space onto a lower order resolution space, and onto its orthogonal complement. The definition of the Discrete Wavelet Transform as a change of coordinates, and how this can be written down from relations between basis functions.

Section 5.3

Definition of the m-level Discrete Wavelet Transform. Matlab implementation of the Haar wavelet transform and its inverse. Experimentation with wavelets on sound.

Section 5.4

Definition of scaling function, mother wavelet, resolution spaces, and detail spaces for the wavelet of piecewise linear functions.

Section 5.5

How one alters the mother wavelet for piecewise linear functions, in order to add a vanishing moment.

Section 5.6

Definition of a multiresolution analysis.

Section 6.1

How one can find the filters of a wavelet transformation by considering its matrix and its inverse. How one can implement the DWT and the IDWT with the help of these filters. Plot of the frequency responses for the filters of the wavelets we have considered, and their interpretation as lowpass and highpass filters.

Section 6.3

How you can call functions which implement the DWT and the IDWT using filter coefficients as input, and use these to experiment on sound.

Section 9.2

How to read, write, and show images with Matlab. How to extract different colour components, convert from colour to grey-level images, and use functions for adjusting the contrast.

Section 9.3

The operation $X \to S_1 X (S_2)^T$ can be used to define operations on images, based on one-dimensional operations S_1 and S_2. This amounts to applying S_1 to all columns in the image, and then S_2 to all rows in the result. You should know how this operation can be conveniently expressed with tensor product notation, and that in the typical case when S_1 and S_2 are filters, this corresponds to a computational molecule, and how to find this molecule. The case when the S_i are smoothing filters gives rise to smoothing operations on images. A simple highpass filter, corresponding to taking the derivative, gives rise to edge-detection operations on images.

Section 9.4

The operation $X \rightarrow S_1 X (S_2)^T$ can also be used to facilitate change of coordinates in images, in addition to filtering images. In other words, change of coordinates is done first column by column, then row by row. The DCT and the DFT are particular changes of coordinates used for images.

Section 10.2

The special interpretation of DWT2 applied to an image as splitting into four types of coordinates (each being one corner of the image), which represent lowpass/highpass combinations in the horizontal/vertical directions.

Section 10.3

You should be able to call functions which performs different wavelet transformations on an image, and be able to interpret the detail components and low-resolution approximations in what you see.

Part III

Chapter 1

First and second order expression for Taylor's formula in several variables. How to compute the gradient and Hessian of linear or quadratic functions defined on $\mathbb{R}^n$.

Chapter 2

The definition of a convex set and a convex function. Simple operations which preserve convex functions (such as that the compositions of an affine and a convex function is convex).

Chapter 3

Nwton's method

Chapter 4

The choice of search direction for the steepest descent method and Newton's method. Armijo's rule for choosing step length (backtracking line search).

Chapter 5

You should know how to state the KKT conditions. The definition of regular points for problems with equalities and/or inequalites. How to traverse the different combinations of active equalities. Positivity of the Lagrange variables μ_j.

Chapter 6

Newton's method for problems with equality constraints. You should be able to write down the barrier function of an optimization problem with inequalities, and write down the barrier problem.